RANDOM FIELDS: GEOMETRIC AND ERGODIC PROPERTIES

MERCREDI 26 JUIN 2013
SALLE DU CONSEIL, AILE TURING, 7TH FLOOR
MAP5, UNIVERSITÉ PARIS DESCARTES
45 RUE DES SAINT PÈRES
75006 PARIS

Program

10h-10h30: Introduction to the workshop (with coffee at Loungex).

10h30-11h30: **Yizao Wang** (University of Cincinnati)
Topics on stationary sum-stable random fields: decomposability and ergodicity.

11h30-12h30: **Clément Dombry** (Université de Poitiers)
Strong mixing properties of max-infinitely divisible random fields.

Lunch (buffet)

14h00-15h00: **Gennady Samorodnitsky** (Cornell University)
Excursion sets over high levels of non-Gaussian infinitely divisible random fields: extreme values, algebra, and geometry.

15h00-16h00: **Laurent Decreusefond** (Télécom ParisTech)
Some geometrical aspects of wireless networks.

16h-16h30: End of the workshop (with coffee at Loungex).

Registration

Registration is free but mandatory. Please send an email to hermine.bierme@mi.parisdescartes.fr before June, 14th.

Organizers:

- Hermine Biermé (MAP5, Université Paris Descartes) hermine.bierme@mi.parisdescartes.fr
- Céline Lacaux (IECL, École des Mines Nancy), celine.lacaux@univ-lorraine.fr

Abstracts

Yizao Wang, University of Cincinnati

Topics on stationary sum-stable random fields: decomposability and ergodicity

Joint work with Parthanil Roy and Stilian A. Stoev

We provide a brief introduction of symmetric alpha-stable (SαS) random fields, and focus on the non-Gaussian ones. We overview some recent developments on the representation and decomposition of stationary symmetric alpha-stable random fields. In particular, we characterize all possible independent SαS components of an SαS process, $0 < \alpha < 2$. We also talk about how the ergodic properties of the stationary SαS random fields are related to the underlying flow/group actions.

Clément Dombry, Université de Poitiers

Strong mixing properties of max-infinitely divisible random fields

Joint work with Frédéric Eyi-Minko

Let $\eta = (\eta(t))_{t\in T}$ be a sample continuous max-infinitely random field on a locally compact metric space T. For a closed subset $S \subset T$, we denote by η_S the restriction of η to S. We consider $\beta(S_1, S_2)$, the absolute regularity coefficient between η_{S_1} and η_{S_2}, where S_1, S_2 are two disjoint closed subsets of T. Our main result is a simple upper bound for $\beta(S_1, S_2)$ involving the exponent measure μ of η: we prove that $\beta(S_1, S_2) \leq 2 \int \mathbb{P}[\eta \not<_{S_1} f,\ \eta \not<_{S_2} f]\, \mu(df)$, where $f \not<_S g$ means that there exists $s \in S$ such that $f(s) \geq g(s)$. If η is a simple max-stable random field, the upper bound is related to the so-called extremal coefficients: for countable disjoint sets S_1 and S_2, we obtain $\beta(S_1, S_2) \leq 4 \sum_{(s_1,s_2)\in S_1\times S_2} (2 - \theta(s_1, s_2))$, where $\theta(s_1, s_2)$ is the pair extremal coefficient. As an application, we show that these new estimates entail a central limit theorem for stationary max-infinitely divisible random fields on $\mathbb{Z}^d$. In the stationary max-stable case, we derive the asymptotic normality of three simple estimators of the pair extremal coefficient.

Gennady Samorodnitsky, Cornell University

Excursion sets over high levels of non-Gaussian infinitely divisible random fields: extreme values, algebra, and geometry

Joint work with Robert J. Adler and Jonathan E. Taylor

We consider smooth, infinitely divisible random fields with regularly varying Lévy measure, and are interested in the geometric characteristics of the excursion sets over high levels u.

For a large class of such random fields we compute the asymptotic joint distribution of the numbers of critical points, of various types, of the random field in the excursion set, conditional on the latter being non-empty. This allows us, for example, to obtain the asymptotic conditional distribution of the Euler characteristic of the excursion set.

In a significant departure from the Gaussian situation, the high level excursion sets for these random fields can have quite a complicated geometry. Whereas in the Gaussian case non-empty excursion sets are, with high probability, roughly ellipsoidal, in the more general infinitely divisible setting almost any shape is possible.

Laurent Decreusefond, Télécom ParisTech

Some geometrical aspects of wireless networks

Joint work with Eduardo Ferraz, Hugues Randriam and Anaïs Vergne

Given a Poisson process on a d-dimensional torus, its random geometric simplicial complex is the complex whose vertices are the points of the Poisson process and simplices are given by the Čech complex associated to the coverage of each point. By means of Malliavin calculus, we compute explicitly the nth order moment of the number of k-simplices. The two first order moments of this quantity allow us to find the mean and the variance of the Euler caracteristic. Also, we show that the number of any connected geometric simplicial complex converges to the Gaussian law when the intensity of the Poisson point process tends to infinity. We use a concentration inequality to find bounds for the for the distribution of the Betti number of first order and the Euler characteristic in such simplicial complex.