On the Revision of Argumentation Systems: Minimal Change of Arguments Statuses

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Introduction

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  - Different semantics: Complete, Stable, Preferred, etc.
- The aim is to know whether an argument is accepted or refused (w.r.t. the chosen semantics σ).
  - An argument $\alpha \in \mathcal{A}$ is (skeptically) accepted iff it belongs to every extension of the argumentation framework for the chosen semantics σ:

$$AF \mid\!\sim_{\sigma} \alpha$$

AGM Framework for Belief Change

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  [Alchourrón, Gärdenfors and Makinson 1985, Katsuno and Mendelzon 1991]
- Representation theorem: "An operator satisfies the postulates iff it is an instance of a given class."

Belief Change Operations [Gärdenfors 1988]

- A formula α can have three different epistemic statuses in the belief base B of an agent:
  - $B \vdash \alpha$: the agent accepts the information α
  - $B \vdash \neg\alpha$: the agent refuses the information α
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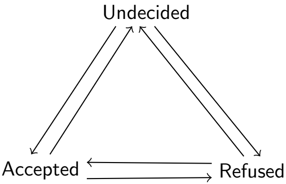

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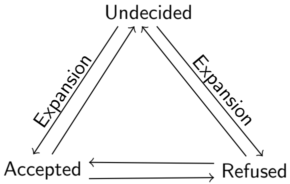

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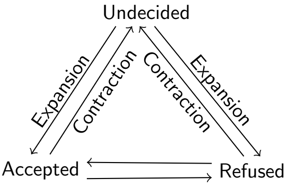

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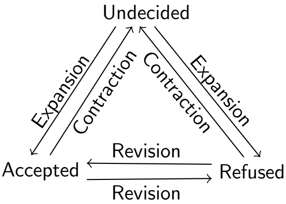

Dynamics of Abstract Argumentation

- Two components of an argument framework:

  Arguments Statuses   Attacks

- Question: What are the fundamental pieces of information for argumentation?
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    - Enforcement [Baumann, Brewka 2010],[Baumann 2012]

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Definition of the Revision of Argumentation Systems

Revision Formulae

- Formulae on the Arguments

$$\Phi ::= \alpha | \neg \Phi | \Phi \wedge \Phi | \Phi \vee \Phi$$

- Candidate Extension
  A *candidate* or *candidate extension* (CE) is a set of arguments.
- Satisfaction of a Formula by a CE
  - $\varepsilon \mid\!\sim \alpha$ iff $\alpha \in \varepsilon$
  - $\varepsilon \mid\!\sim \neg\varphi$ iff $\varepsilon \not\mid\!\sim \varphi$
  - $\varepsilon \mid\!\sim \varphi \wedge \psi$ iff $\varepsilon \mid\!\sim \varphi$ and $\varepsilon \mid\!\sim \psi$
  - $\varepsilon \mid\!\sim \varphi \vee \psi$ iff $\varepsilon \mid\!\sim \varphi$ or $\varepsilon \mid\!\sim \psi$
- Satisfaction of a Formula by an Argumentation Framework

$$AF \mid\!\sim_{} \varphi \text{ iff } \forall \varepsilon \in Ext_{}(AF), \varepsilon \mid\!\sim \varphi$$

Postulates Expressed with the Extensions

Notation

A_φ denotes the set of the CE which satisfy φ.

Postulates

- **(AE1)** $Ext(AF \star \varphi) \subseteq A_\varphi$
- **(AE2)** If $Ext(AF) \cap A_\varphi \neq \emptyset$ then $Ext(AF \star \varphi) = Ext(AF) \cap A_\varphi$
- **(AE3)** If φ is consistent, then $Ext(AF \star \varphi) \neq \emptyset$
- **(AE4)** If $\mathcal{A}_\varphi = \mathcal{A}_\psi$, then $Ext_\psi(AF \star \varphi) = Ext_\psi(AF \star \psi)$
- **(AE5)** $Ext(AF \star \varphi) \cap A_\psi \subseteq Ext(AF \star \varphi \wedge \psi)$
- **(AE6)** If $Ext(AF \star \varphi) \cap A_\psi \neq \emptyset$ then $Ext(AF \star \varphi \wedge \psi) \subseteq Ext(AF \star \varphi) \cap A_\psi$

Representation Theorem (1)

A faithful assignment is a mapping from an argumentation framework $AF = \langle A, R \rangle$ (given a semantics σ) to a total pre-order $\leqslant_{AF}$ on the set of CE s.t.:

- if $\varepsilon_1 \in Ext\ (AF)$ and $\varepsilon_2 \in Ext\ (AF)$, then $\varepsilon_1 \simeq_{AF} \varepsilon_2$
- if $\varepsilon_1 \in Ext\ (AF)$ and $\varepsilon_2 \notin Ext\ (AF)$, then $\varepsilon_1 <_{AF} \varepsilon_2$

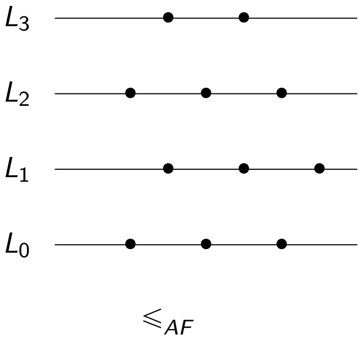
Representation Theorem (2)

Theorem

Given a semantics σ, a revision operator $\star$ satisfies the rationality postulates **(AE1)-(AE6)** iff there exists a faithful assignment which maps every argumentation framework $AF = \langle A, R \rangle$ to a total pre-order $\leqslant_{AF}$ s.t.:

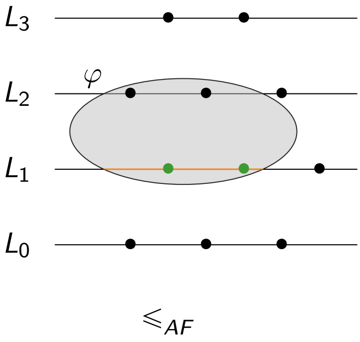
$$Ext\ (AF \star \varphi) = \min(A_{\varphi}, \leqslant_{AF})$$

Example of Pre-Order



- Every point represents a CE
- Level L_0 = σ-extensions of AF
- Other levels = other CEs sorted by "distance"

Choice of Minimal CE



- Shaded area: A ,
- Green points: minimal elements of A ,

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A Two-step Process

Distance-based Revision Operators (1)

- Pre-order between CE
  Let AF be an argumentation framework and σ be a semantics. Given d a distance between CE, one defines $\leqslant_{AF}^{;d}$ by

$$\varepsilon \leqslant_{AF}^{;d} \varepsilon' \text{ iff } d(\varepsilon, Ext\ \ (AF)) \leqslant d(\varepsilon', Ext\ \ (AF))$$

- Example of distance: Hamming Distance
  - $d_H(\varepsilon, \varepsilon') = |(\varepsilon \backslash \varepsilon') \cup (\varepsilon' \backslash \varepsilon)|$
  - $d_H(\varepsilon, \{\varepsilon'_1, \dots, \varepsilon'_n\}) = \min_{1 \leqslant i \leqslant n} d_H(\varepsilon, \varepsilon'_i)$

Distance-based Revision Operators (2)

- Distance-based Revision Operator
  Let σ be a semantics, and d be a distance between CE.
  The distance-based operator $\star^d$ is defined as

$$Ext\ (AF \star^d \varphi) = \min(A_{'}, \leqslant_{AF}^{;d})$$

- Every distance-based operator satisfies the postulates **(AE1)**-**(AE6)**.

Example

- Framework to revise

$$AF = \textcircled{a} \quad \textcircled{b} \quad \textcircled{c} \qquad\qquad \varphi = (a \vee b) \wedge (\neg a \vee \neg b)$$

- Its (single) stable extension
  - $\{a, b, c\}$
- Revised extensions
  - $Ext_{st}(AF \star \varphi) = \{\{a, c\}, \{b, c\}\}$

Generation of Corresponding Argumentation Frameworks

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  - Generation Operator
    A generation operator $\mathcal{AF}$ is a mapping from a set of CE $\mathcal{C}$ to a set of argumentation frameworks s.t. $Ext\ (\mathcal{AF}\ (\mathcal{C})) = \mathcal{C}$.
  - Basic Revision Operator

$$AF \star \varphi = \mathcal{AF}\ (\min(A_{'}, \leqslant_{AF}))$$

$\rightarrow$ satisfies the postulates.

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$$AF \longrightarrow \text{extensions} \longrightarrow AFs$$

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- Minimal change of the attack relation
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Minimal change in the revision step: arguments statuses

What is minimality in the generation step?

- Minimal change of the attack relation
- Minimal cardinality of the result
- Combination of both

Minimal Change of the Attack Relation

- dg: a distance between graphs
- $\mathcal{C}$: a set of CE
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Example (2)

AF 

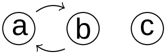
- ' p a _ bq ^ p a _ bq
- Ext_{st} pAF ' q tt a; cu; t b; cuu

Minimal change of attack relation

AF_1

AF_2

Minimal cardinality



AF_3