Merging of Abstract Argumentation Frameworks

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15th International Conference on Principles of Knowledge Representation and Reasoning
25 – 29 April 2016

Database and Artificial Intelligence Group

Outline

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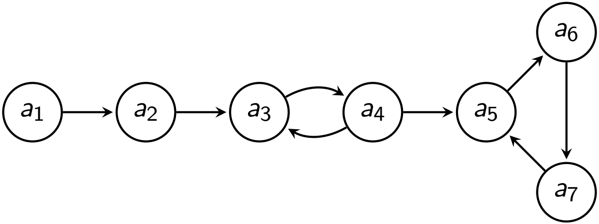
Abstract AF [Dung, AIJ 1995]

- An AF is a digraph $F = \langle A, R \rangle$, A is the set of arguments and $R \subseteq A \times A$ is the attack relation
- Evaluation of arguments: Many semantics to compute extensions
  - grounded, stable, preferred, complete,...



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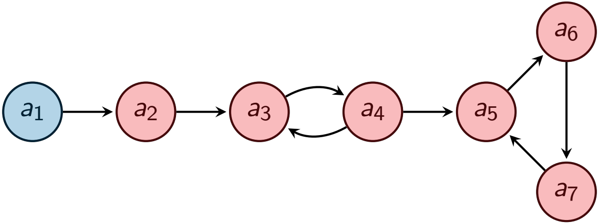

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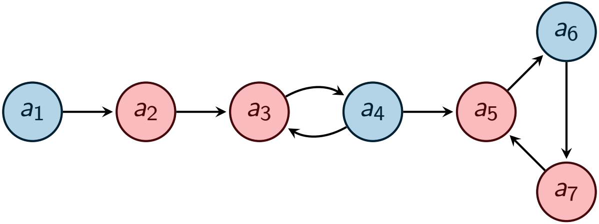

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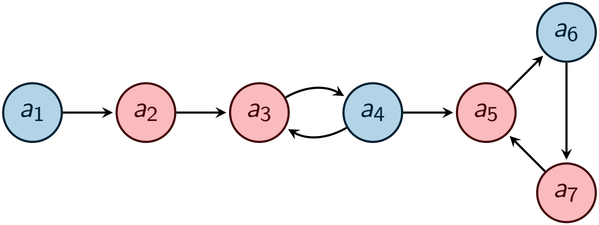

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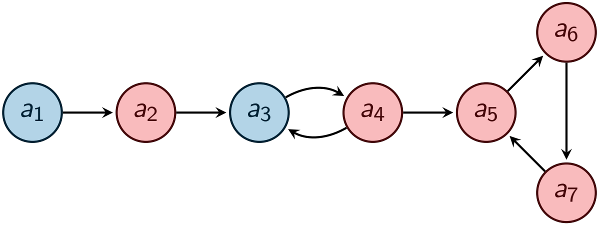

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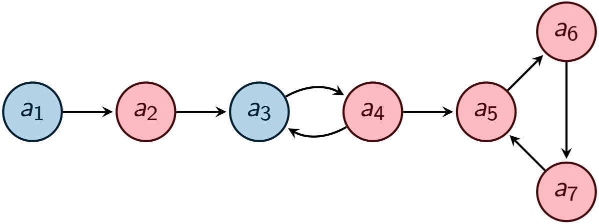

$Ext_{co}(F) = \{\{a_1, a_4, a_6\}, \{a_1, a_3\}, \{a_1\}\}$

Revision of AFs

[Coste et al, KR 2014]

- Revision of an AF F by a formula φ which expresses conditions on extensions
- A two-step process:

$$\text{Inputs} \quad \left.\begin{matrix} F \\ \varphi \end{matrix}\right\} \Longrightarrow \text{revised extensions} \Longrightarrow \underset{\text{Outputs}}{\{F'_1, \ldots, F'_k\}}$$

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[Diller et al, IJCAI 2015]

- Modification of rationality postulates: result is required to be a single AF [Dunne et al, AIJ 2015]

$$\begin{array}{ccccc} \text{Inputs} & & & & \text{Outputs} \\ \left.\begin{array}{c} F \\ \varphi \end{array}\right\} & \Longrightarrow & \textbf{realizable}\ \text{revised extensions} & \Longrightarrow & F' \end{array}$$

Outline

Schematic Explanation

- Merging of a profile of AFs $\langle F_1, \ldots, F_n \rangle$, with an integrity constraint μ which expresses conditions on extensions
- A two-step process:

$$\begin{array}{ccccc} \text{Inputs} & & & & \text{Outputs} \\ \left.\begin{array}{l} \langle F_1, \ldots, F_n \rangle \\ \mu \end{array}\right\} & \Longrightarrow & \text{extensions} & \Longrightarrow & \{F'_1, \ldots, F'_k\} \end{array}$$

Questions:

- How to obtain the extensions?
- How to obtain the AFs?

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Rationality Postulates

Postulates adapted from propositional belief merging [Konieczny and Pino Pérez, JLC 2002]

(M0) $Ext_\sigma(\Delta_\mu(\mathcal{F})) \subseteq \mathcal{A}^\sigma_\mu$

(M1) If $\mathcal{A}^\sigma_\mu \neq \emptyset$, then $Ext_\sigma(\Delta_\mu(\mathcal{F})) \neq \emptyset$

(M2) If $Ext_\sigma(\bigwedge\!\!\bigwedge \mathcal{F}) \cap \mathcal{A}^\sigma_\mu \neq \emptyset$, then $Ext_\sigma(\Delta_\mu(\mathcal{F})) = Ext_\sigma(\bigwedge\!\!\bigwedge \mathcal{F}) \cap \mathcal{A}^\sigma_\mu$

(M3) If $\mathcal{F}_1 \equiv \mathcal{F}_2$ and $\mu_1 \equiv_\sigma \mu_2$, then $Ext_\sigma(\Delta_{\mu_1}(\mathcal{F}_1)) = Ext_\sigma(\Delta_{\mu_2}(\mathcal{F}_1))$

(M4) If $Ext_\sigma(F_1) \subseteq \mathcal{A}^\sigma_\mu$ and $Ext_\sigma(F_2) \subseteq \mathcal{A}^\sigma_\mu$, then
$Ext_\sigma(\Delta_\mu(\langle F_1, F_2\rangle)) \cap Ext_\sigma(F_1) \neq \emptyset$ implies
$Ext_\sigma(\Delta_\mu(\langle F_1, F_2\rangle)) \cap Ext_\sigma(F_2) \neq \emptyset$

(M5) $Ext_\sigma(\Delta_\mu(\mathcal{F}_1)) \cap Ext_\sigma(\Delta_\mu(\mathcal{F}_2)) \subseteq Ext_\sigma(\Delta_\mu(\mathcal{F}_1 \cup \mathcal{F}_2))$

(M6) If $Ext_\sigma(\Delta_\mu(\mathcal{F}_1)) \cap Ext_\sigma(\Delta_\mu(\mathcal{F}_2)) \neq \emptyset$, then
$Ext_\sigma(\Delta_\mu(\mathcal{F}_1 \cup \mathcal{F}_2)) \subseteq Ext_\sigma(\Delta_\mu(\mathcal{F}_1)) \cap Ext_\sigma(\Delta_\mu(\mathcal{F}_2))$

(M7) $Ext_\sigma(\Delta_{\mu_1}(\mathcal{F})) \cap \mathcal{A}^\sigma_{\mu_2} \subseteq Ext_\sigma(\Delta_{\mu_1 \wedge \mu_2}(\mathcal{F}))$

(M8) If $Ext_\sigma(\Delta_{\mu_1}(\mathcal{F})) \cap \mathcal{A}^\sigma_{\mu_2} \neq \emptyset$, then $Ext_\sigma(\Delta_{\mu_1 \wedge \mu_2}(\mathcal{F})) \subseteq Ext_\sigma(\Delta_{\mu_1}(\mathcal{F})) \cap \mathcal{A}^\sigma_{\mu_2}$

Representation Theorem

Syncretic Assignment

Mapping from any profile $\mathcal{F}$ to a total pre-order on extensions $\leqslant_{\mathcal{F}}$ s.t.

1. If $c_1 \in Ext_\sigma(\bigwedge\!\!\!\bigwedge \mathcal{F})$, $c_2 \in Ext_\sigma(\bigwedge\!\!\!\bigwedge \mathcal{F})$, then $c_1 \simeq_{\mathcal{F}} c_2$
2. If $c_1 \in Ext_\sigma(\bigwedge\!\!\!\bigwedge \mathcal{F})$, $c_2 \notin Ext_\sigma(\bigwedge\!\!\!\bigwedge \mathcal{F})$, then $c_1 <_{\mathcal{F}} c_2$
3. $\forall c_1 \in Ext_\sigma(F_1), \exists c_2 \in Ext_\sigma(F_2)$ s.t. $c_2 \leqslant_{(F_1,F_2)} c_1$
4. If $c_1 \leqslant_{\mathcal{F}_1} c_2$ and $c_1 \leqslant_{\mathcal{F}_2} c_2$, then $c_1 \leqslant_{\mathcal{F}_1 \cup \mathcal{F}_2} c_2$
5. If $c_1 <_{\mathcal{F}_1} c_2$ and $c_1 \leqslant_{\mathcal{F}_2} c_2$, then $c_1 <_{\mathcal{F}_1 \cup \mathcal{F}_2} c_2$

Theorem

Δ satisfies **(M0)-(M8)** iff $Ext_\sigma(\Delta_\mu(\mathcal{F})) = \min(\mathcal{A}_\mu^\sigma, \leqslant_{\mathcal{F}})$

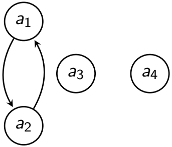
Distance-based Merging

- d: distance between sets of arguments (e.g. Hamming distance)
- $\otimes$: aggregation function (e.g. sum)
- $\mathcal{F} \mapsto \leqslant_{\mathcal{F}}^{\otimes,d}$: syncretic assignment defined by

$$c_1 \leqslant_{\mathcal{F}}^{\otimes,d} c_2 \text{ iff } \otimes_{F\in\mathcal{F}}\, d(c_1, Ext_\sigma(F)) \leqslant \otimes_{F\in\mathcal{F}} d(c_2, Ext_\sigma(F))$$

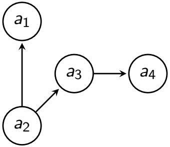
Example of Distance-based Merging

F_1



$Ext_{st}(F_1) = \{\{a_1, a_3, a_4\}, \{a_2, a_3, a_4\}\}$

F_2



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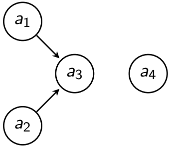
F_3



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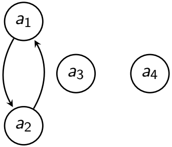

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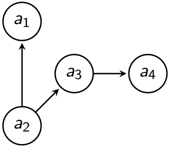

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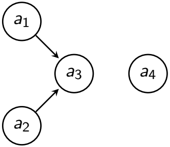

$Ext_{st}(F_3) = \{\{a_1, a_2, a_4\}\}$

Goal: merging $\mathcal{F} = \langle F_1, F_2, F_3 \rangle$ with constraint $\mu = a_2 \wedge a_4 \wedge (a_1 \vee a_3)$

μ	F_1 $\{a_1, a_3, a_4\}$ $\{a_2, a_3, a_4\}$	F_2 $\{a_2, a_4\}$	F_3 $\{a_1, a_2, a_4\}$	Σ
$\{a_1, a_2, a_4\}$	2	1	0	3
$\{a_2, a_3, a_4\}$	0	1	2	3
$\{a_1, a_2, a_3, a_4\}$	1	2	1	4

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Example of Distance-based Merging

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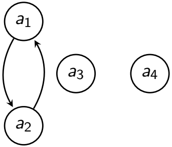

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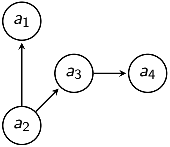

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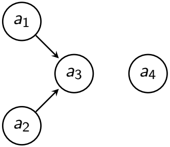

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Reminder: A Two-Step Process

$$\left.\begin{array}{l}\text{Inputs}\\ \langle F_1, \ldots, F_n \rangle \\ \mu \end{array}\right\} \Longrightarrow \text{extensions} \Longrightarrow \begin{array}{c}\text{Outputs}\\ \{F'_1, \ldots, F'_k\}\end{array}$$

- Postulates, representation theorem: selection of extensions
- Generation operators: obtaiting AFs

Reminder: A Two-Step Process

Inputs				Outputs
$\langle F_1, \ldots, F_n \rangle$ μ	$\Longrightarrow$	extensions	$\Longrightarrow$	$\{F'_1, \ldots, F'_k\}$

- Postulates, representation theorem: selection of extensions
- Generation operators: obtaiting AFs

Generation of AFs

- mapping $\mathcal{AF}_\sigma$ from a set of extensions $\mathcal{C}$ to a set of AFs $\mathcal{F}$ s.t. $Ext_\sigma(\mathcal{F}) = \mathcal{C}$.
- Full merging operator: $\mathcal{AF}_\sigma(\min(\mathcal{A}^\sigma_\mu, \leqslant_\mathcal{F}))$
- Two policies to handle minimal change

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- Two policies to handle minimal change

Minimal change of attack, then minimal cardinality	Minimal cardinality, then minimal change of attack

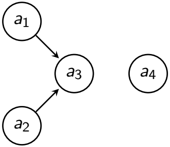
Example of Generation

Reminder: at the first step, we obtained

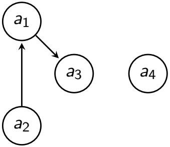
$$Ext_{st}(\Delta_{\mu}^{\sum,d_H}(\langle F_1, F_2, F_3\rangle)) = \{\{a_1, a_2, a_4\}, \{a_2, a_3, a_4\}\}$$

Attack, then cardinality

F'_1

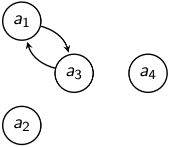


F'_2



Cardinality, then attack

F'_3



Resolute Merging: Schematic Explanation

- Is it possible to represent the group's beliefs by a single AF?
- A two-step process:

Inputs				Outputs
$\langle F_1, \ldots, F_n \rangle$, μ	$\Longrightarrow$	realizable extensions	$\Longrightarrow$	F'

Question:

- Adaption of the first step to obtain realizable extensions?

Resolute Merging: Schematic Explanation

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Question:

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Resolute Merging

- σ-compliant assignment [Diller et al, IJCAI 2015]: pre-order $\leqslant$ s.t. for any formula μ, $\min(A_\mu^\sigma, \leqslant)$ is σ-realizable

Good News

A resolute merging operator satisfies the postulates iff there is a σ-compliant syncretic assignment s.t. $Ext_\sigma(\Delta_\mu(\mathcal{F})) = \min(A_\mu^\sigma, \leqslant)$

Resolute Merging

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Bad News

There are no resolute merging operators for stable, preferred, grounded and complete semantics.

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Outline

FUS_{All}, FUS_{AllNT}, FUS_{MajNT} [Delobelle et al, IJCAI 2015]

$$\text{Inputs } \langle F_1, \ldots, F_n \rangle \implies \text{Weighted AF} \implies \text{Outputs extensions}$$

- no integrity constraint (i.e. $\mu = \top$): **(M0)**,**(M7)**,**(M8)** trivially satisfied

	FUS_{All}	FUS_{AllNT}	FUS_{MajNT}
(M1)	$\times$	$\checkmark$	$\checkmark$
(M2)	$\times$	$\times$	$\times$
(M3)	$\checkmark$	$\checkmark$	$\checkmark$
(M4)	$\times$	$\times$	$\times$
(M5)	$\times$	$\times$	$\times$
(M6)	$\times$	$\times$	$\times$

Aggregation Axioms [Dunne et al, COMMA 2012; Delobelle et al, IJCAI 2015]

- **Anonymity** aggregation is not sensible to permutations of the profile
- **Non-triviality** the result has at least one non-empty extension
- **Decisiveness** the result has exactly one non-empty extension
- **Unanimity** when agents agree on something, it belongs to the result
- **Majority** when most of the agents agree on something, it belongs to the result
- **Closure** everything in the result is in some part of the input
- **Identity** if all AFs are identical, the result is the initial AF

Merging Operators vs Aggregation Axioms

Properties	Σ, dg	$\Sigma, card$	Lex, dg	$Lex, card$
ANON	✓	✓	✓	✓
σ-SNT/σ-WNT	×	×	×	×
σ-SD / σ-WD	×	×	×	×
UA	×	×	×	×
σ-U / sa_σ-U	✓	✓	✓	✓
ca_σ-U	$\checkmark^{gr}$	$\checkmark^{gr}$	$\checkmark^{gr}$	$\checkmark^{gr}$
MAJ-A	×	×	×	×
σ-MAJ / ca_σ-MAJ	$\checkmark^{gr}$	$\checkmark^{gr}$	×	×
sa_σ-MAJ	✓	✓	×	×
CLO / AC / σ-C	×	×	×	×
ca_σ-C	✓	✓	✓	✓
sa_σ-C	$\checkmark^{gr}$	$\checkmark^{gr}$	$\checkmark^{gr}$	$\checkmark^{gr}$
ID	✓	✓	✓	✓

Discussion: Two Different Philosophies of AF Merging

It is not surprising that

- $FUS_{All}, FUS_{AllNT}, FUS_{MajNT}$ do not satisfy many IC-merging postulates
- our merging operators do not satisfy many aggregation axioms

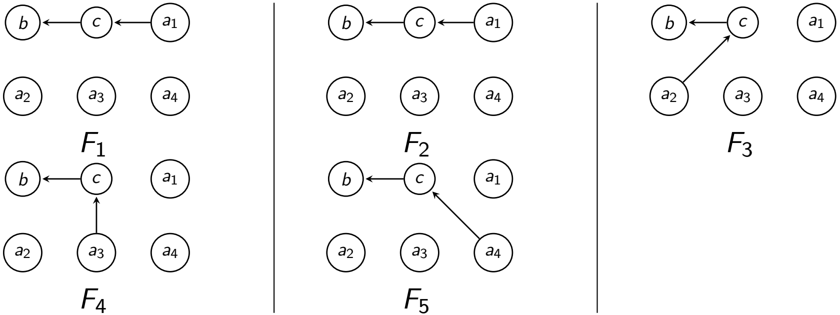
Both approaches follow different intuitions

Operators	$FUS_{All}, FUS_{AllNT}, FUS_{MajNT}$	Δ_μ-family
Properties	Aggregation axioms	IC-Merging Postulates
Information	Attacks	Extensions

Example

Attack-based Merging [Coste-Marquis et al 2007, Tohmé et al 2008, Delobelle et al 2015]

- $\mathcal{F} = \langle F_1, F_2, F_3, F_4, F_5 \rangle$
- Only $c \rightarrow b$ belongs to all AFs



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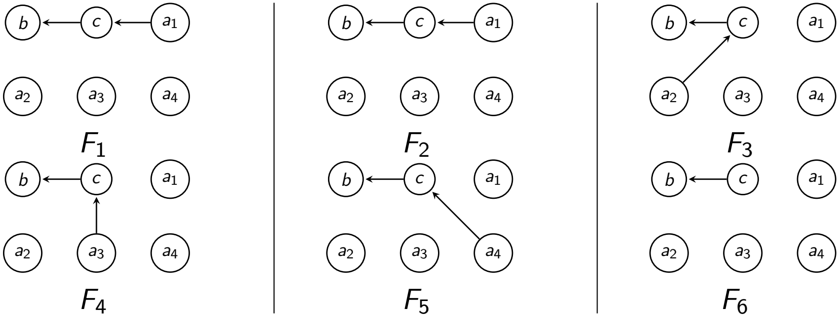
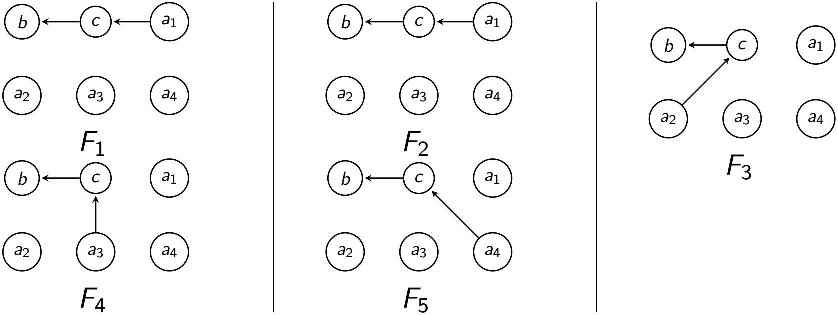
- $\mathcal{F} = \langle F_1, F_2, F_3, F_4, F_5 \rangle$
- Only $c \rightarrow b$ belongs to all AFs
- Result of merging is F_6



Example

Extension-based Merging

- $\mathcal{F} = \langle F_1, F_2, F_3, F_4, F_5 \rangle$
- $\{a_1, a_2, a_3, a_4, b\}$ is the single extension for all AFs: must be selected at first step of merging



Example

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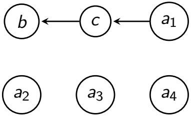
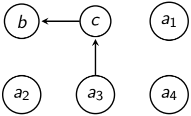
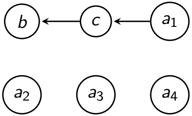
- Result of merging is F_1



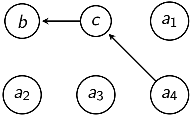
F_1



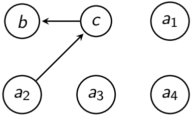
F_4



F_2



F_5



F_3

Since $F_1 = F_2$, F_1 is the AF closest to the profile

Outline

Conclusion

Summary

- New family of AF merging operators, inspired by extension-based revision [Coste et al, KR 2014]
  - Axiomatic characterization + representation theorem
  - Concrete operators: distance-based merging
- New philosophy of AF merging, orthogonal to attack-based merging

Future works

- Determine resolute merging operators similar to resolute revision operators [Diller et al, IJCAI 2015]
- Study other attack-based approaches [Coste-Marquis et al 2007, Tohmé et al 2008]
- Computational aspects and algorithms design