Comp 775: Image Registration

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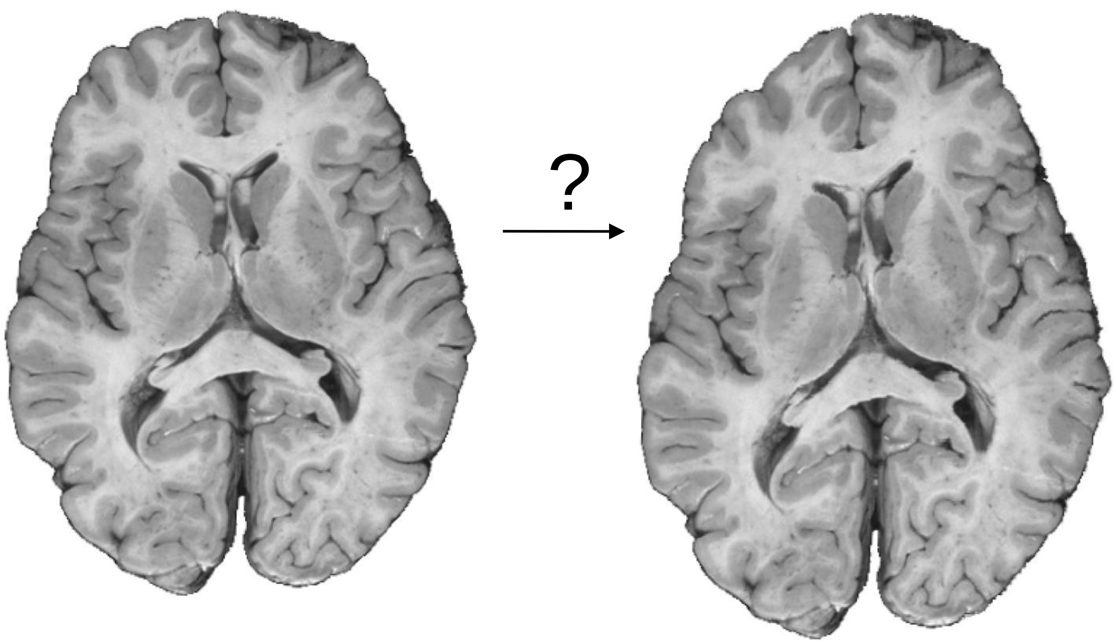
Purpose of Image Registration

Establishing a geometric transformation

$$\underline{x}' = \underline{h}(\underline{x}) = \underline{x}' = \underline{x} + \underline{\Delta x}$$

relating points in one image to points in another.



Source Image

Target Image

Purpose of Image Registration

Establishing a geometric transformation

$$\underline{x}' = \underline{h}(\underline{x}) = \underline{x}' = \underline{x} + \underline{\Delta x}$$

relating points in one image to points in another.

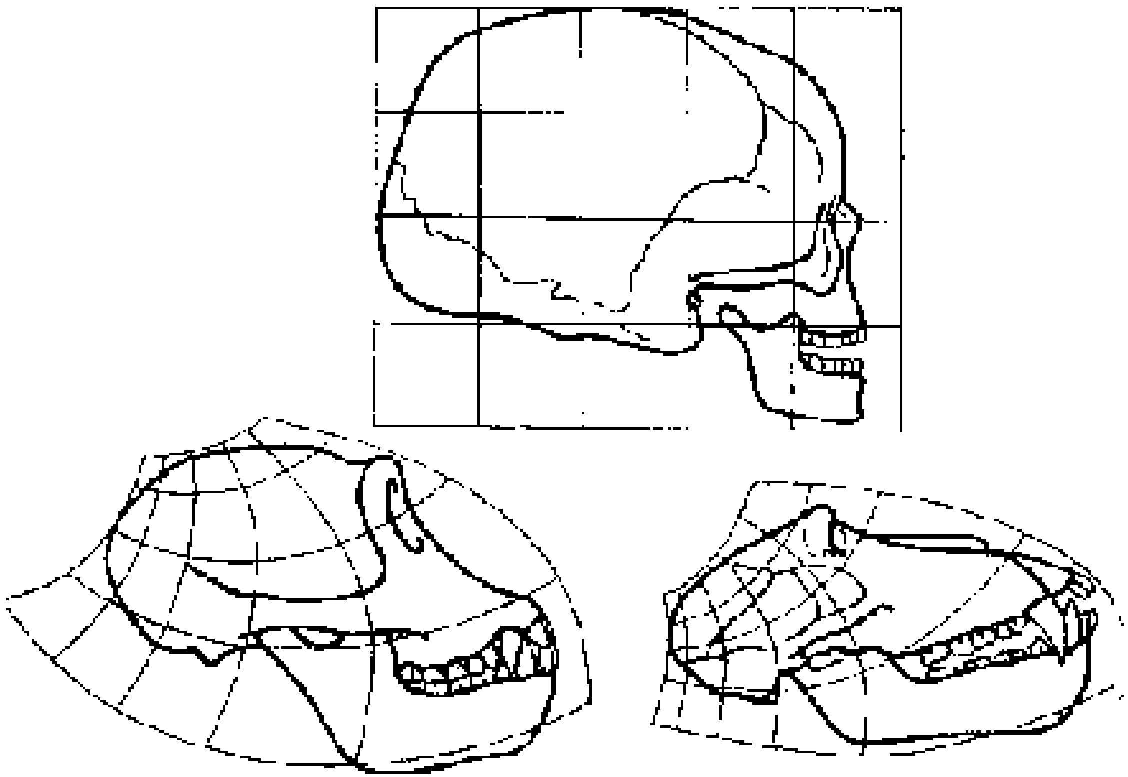
Needed (as with segmentation)

- Regularization Energy (Geometric atypicality)
- Data Similarity Measure (geometry-to-data match)
- Optimization Scheme

And also

- Transformation Model
- Interpolation Model

Registration Example



Skulls of a human, a chimpanzee and a baboon
and transformations between them

D'Arcy Thompson

Coordinate Systems

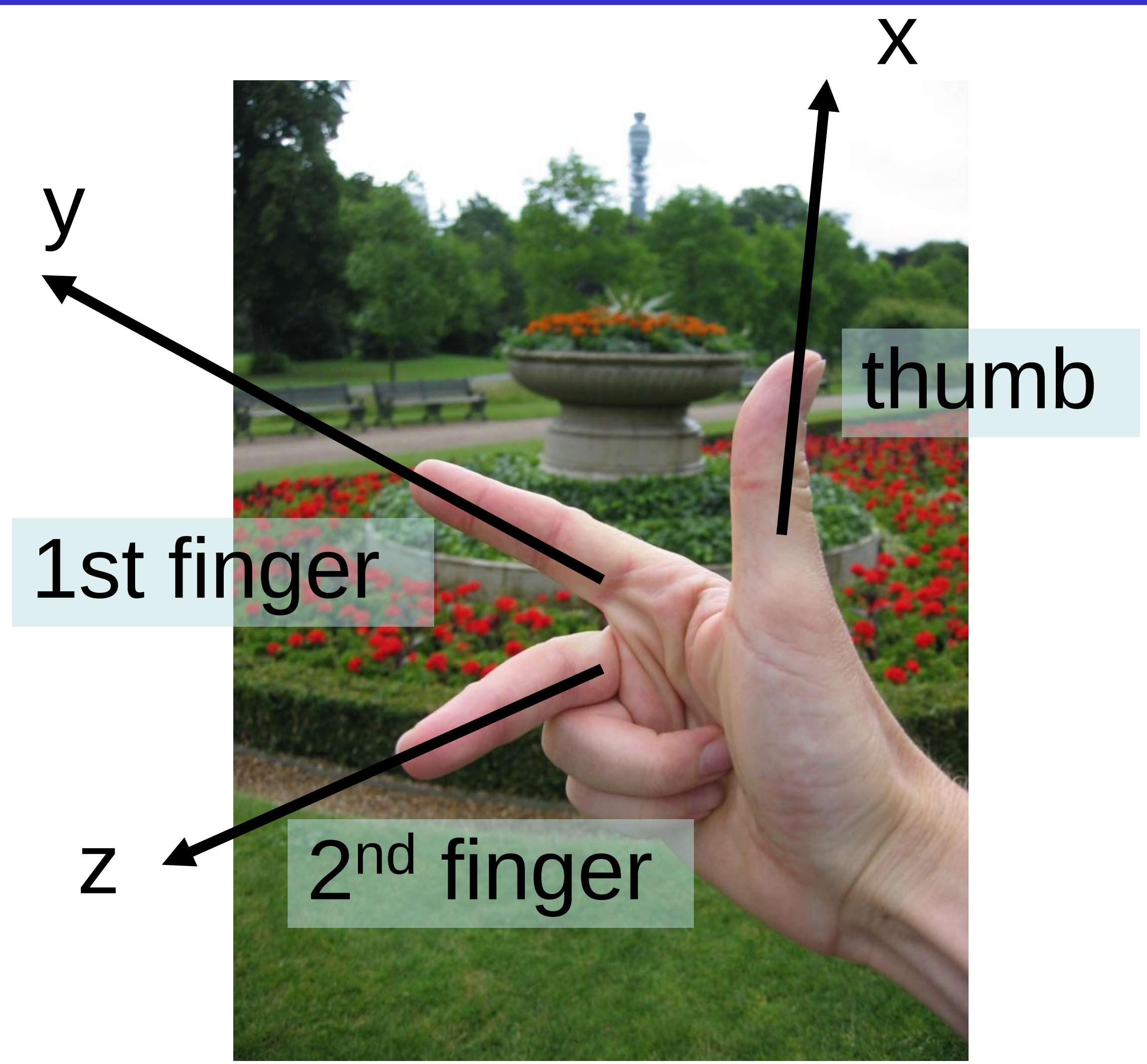
Important: Make sure to get your coordinate systems straight.

- Image data is more than just a 3D-array of numbers
- Images may have been acquired in different coordinates systems
- Orientationally dependent data (e.g., tensors) may have additional coordinate systems.

To be certain the alignment can work properly work in

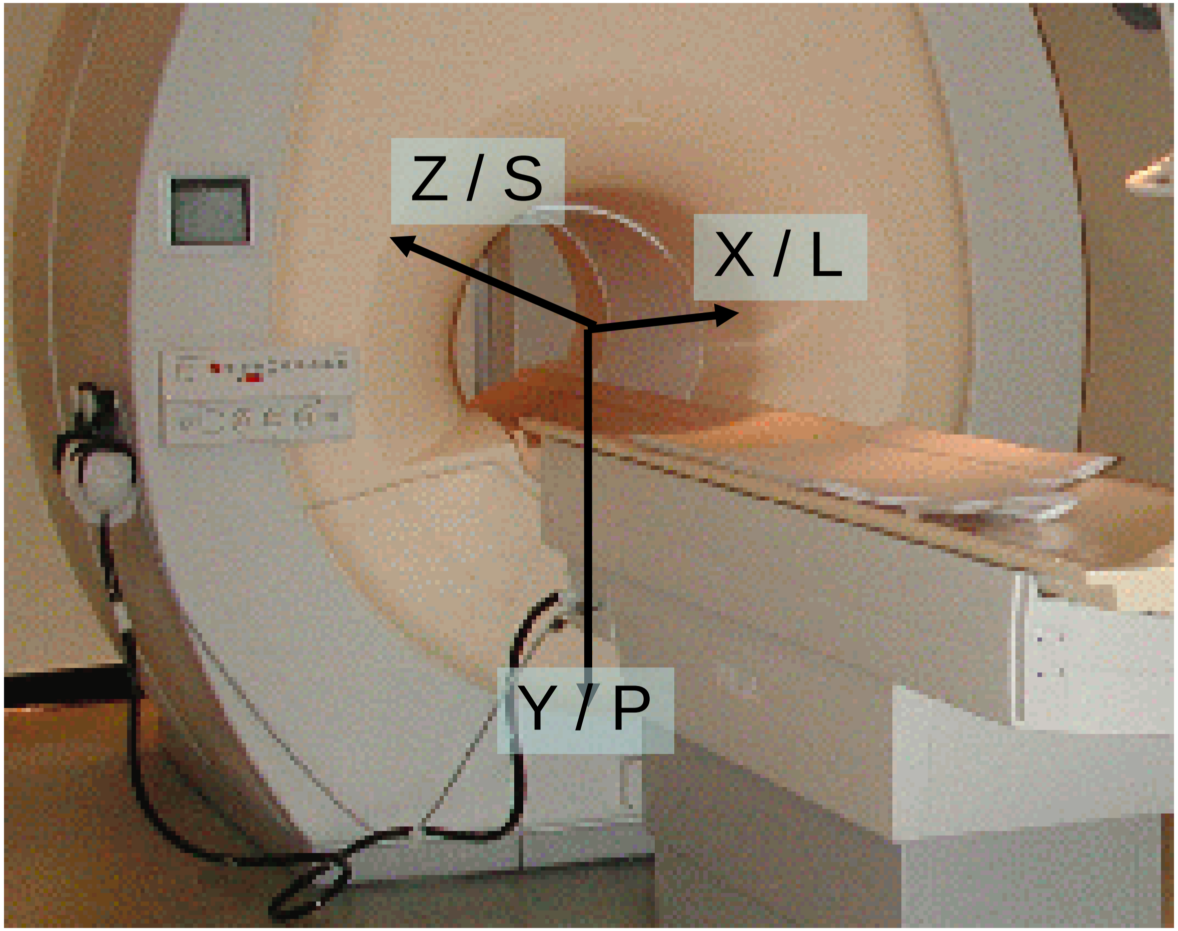
World coordinates

Right-handed coordinate systems



Right handed coordinate system. Image: Atkinson

Coordinate Systems



L: Left
P: Posterior
S: Superior

R: Right
A: Anterior
S: Superior

Supine (face-up) patient.

Image: Atkinson

Coordinate Systems

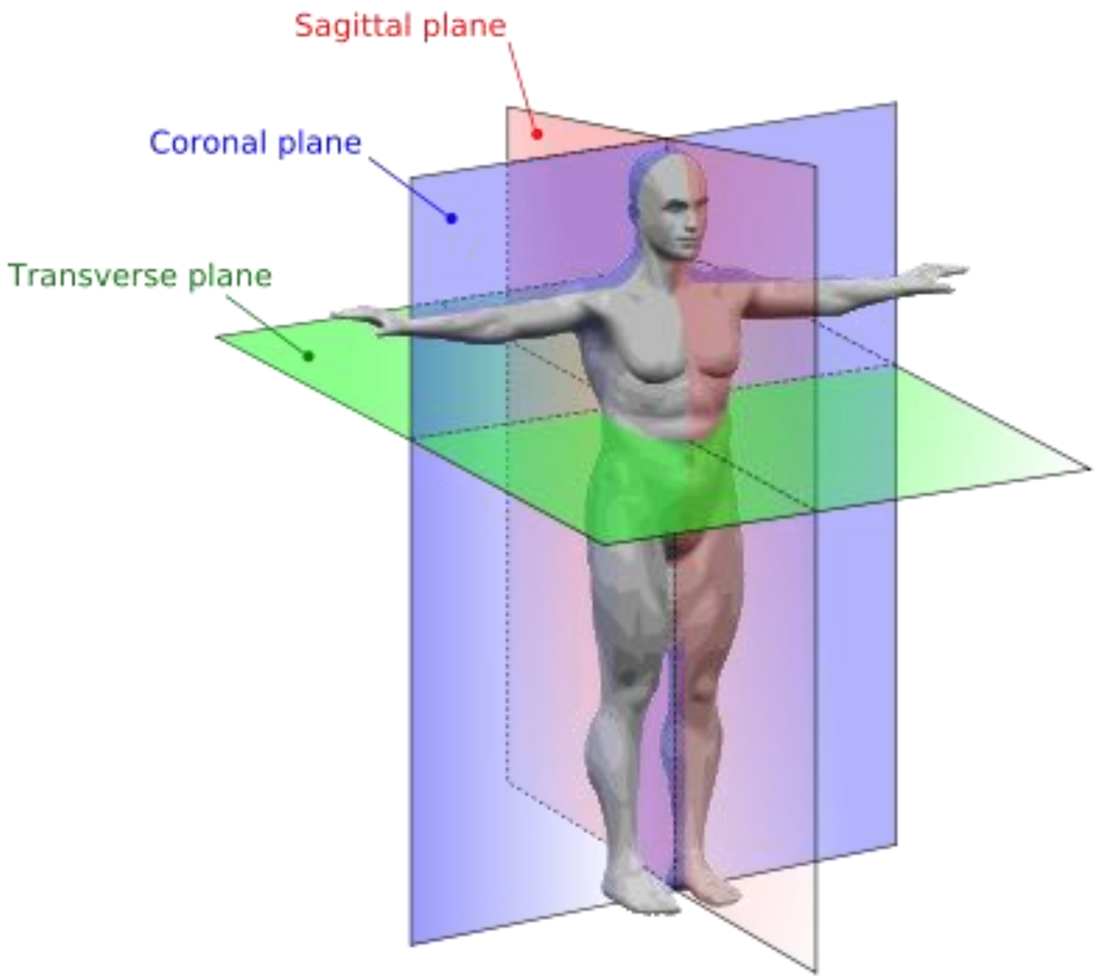
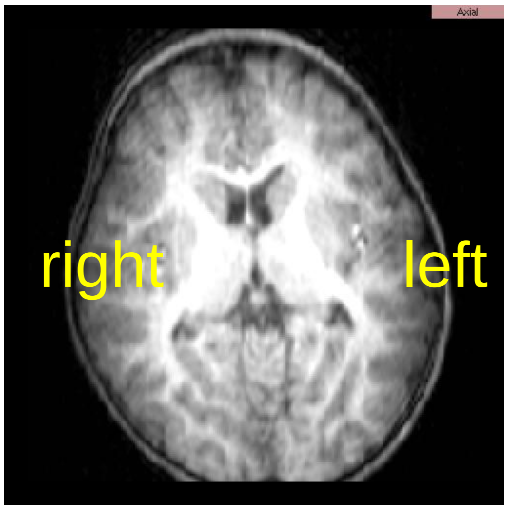


Image:Wikipedia

Coordinate Systems

Axial

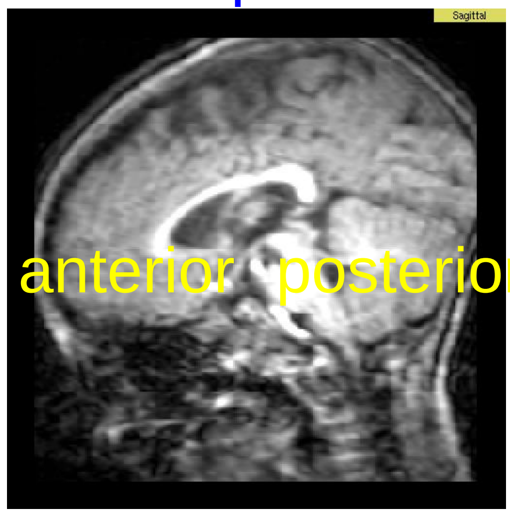
anterior



posterior

Sagittal

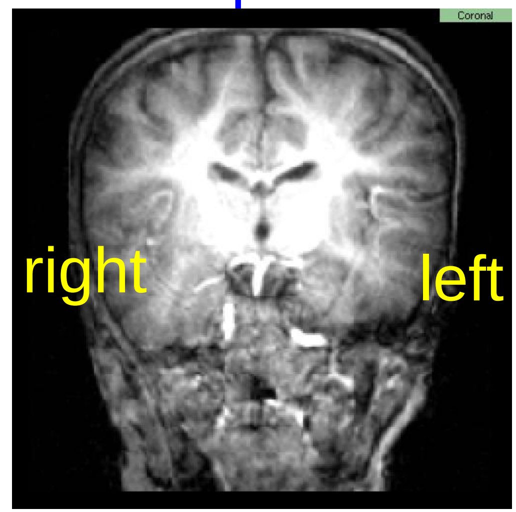
superior



inferior

Coronal

superior



inferior

Important: Make sure to get your coordinate systems straight.
What is right and what is left for example?

Coordinate Systems: Viewpoints

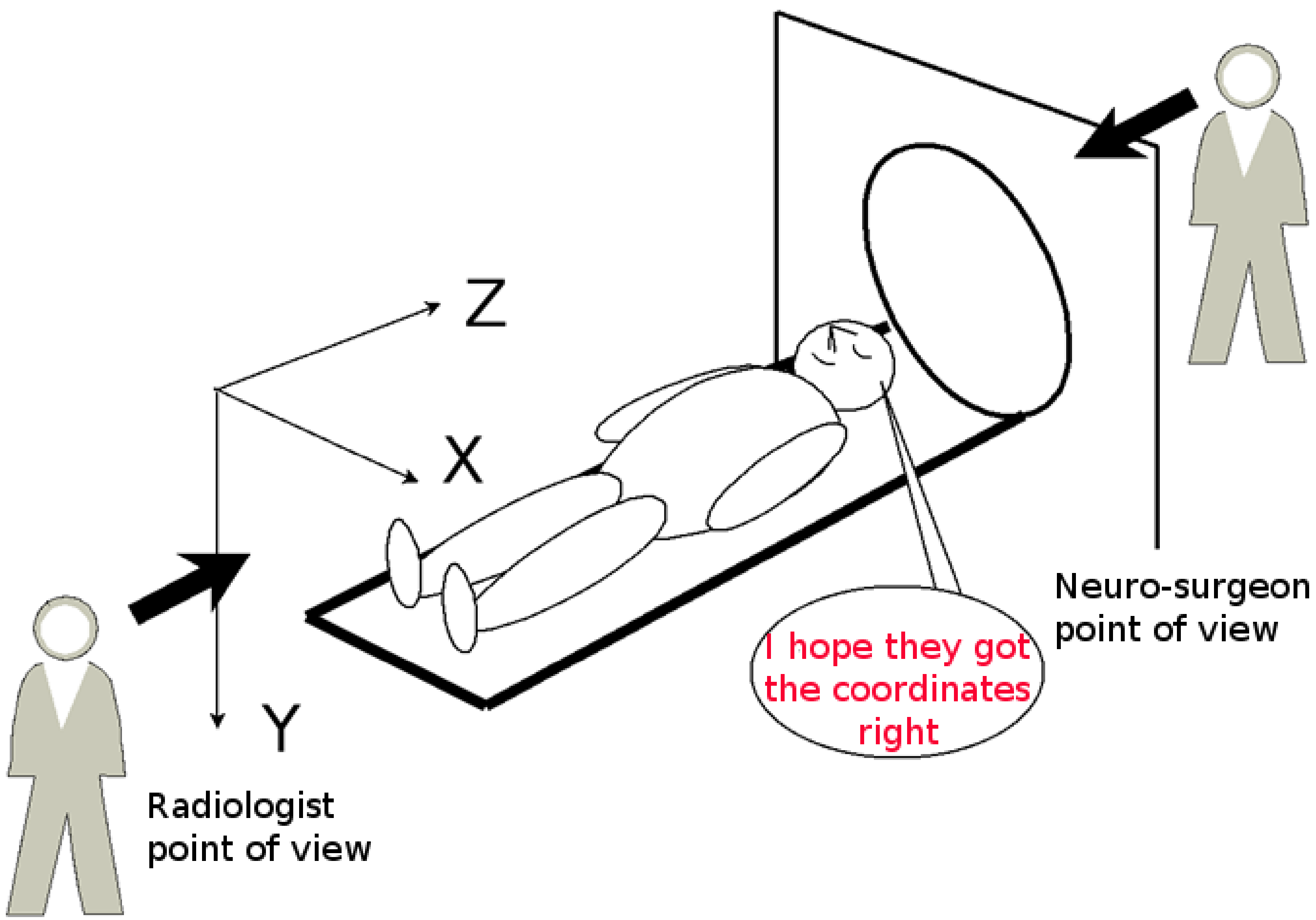


Image: Adapted from Kitware

Pixels/Voxels vs. World Coordinates

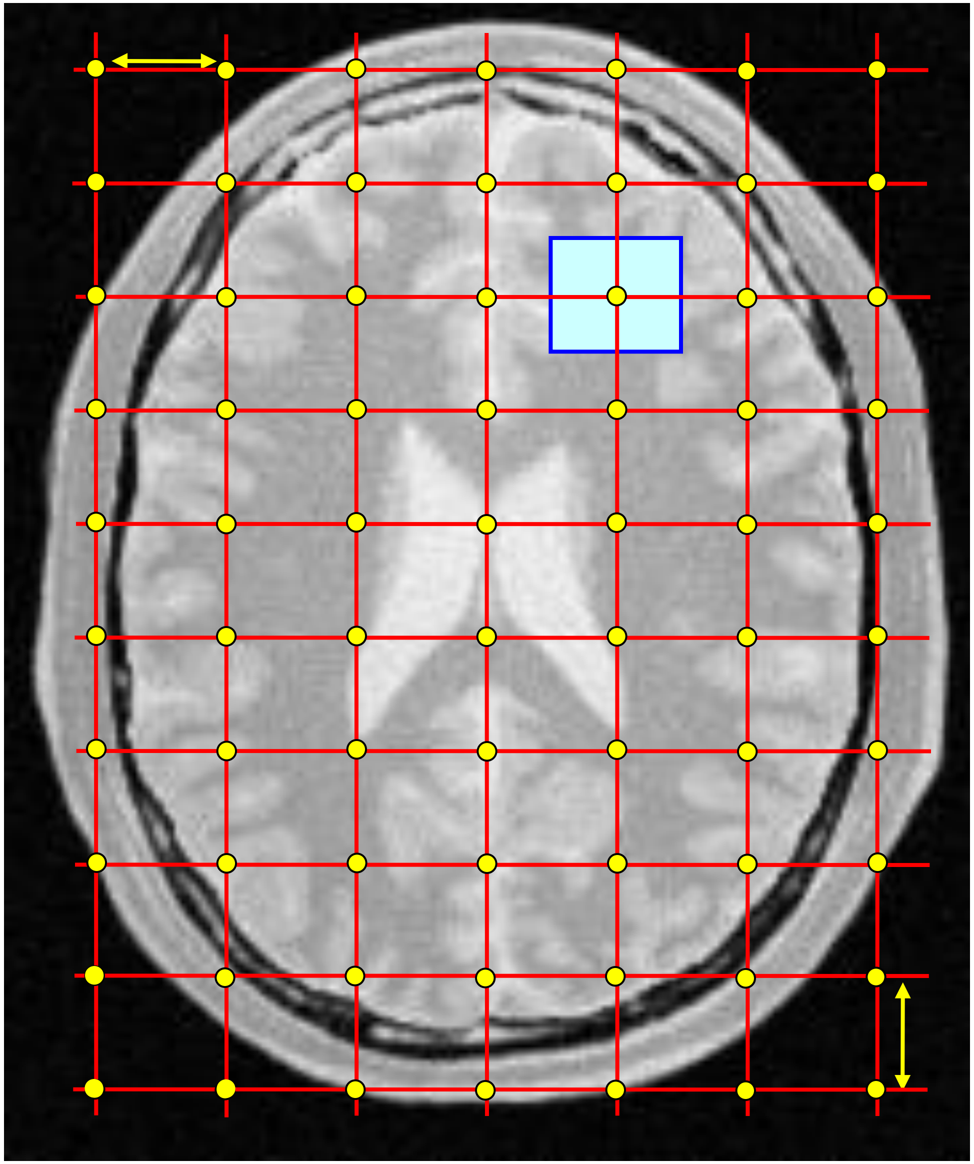
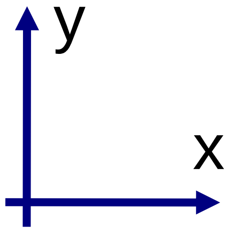


Image: ITK Registration Guide

Image Coordinates

Spacing (Sx)



Origin (Ox,Oy)

Image: ITK Registration Guide

Spacing (Sy)

Image coordinates

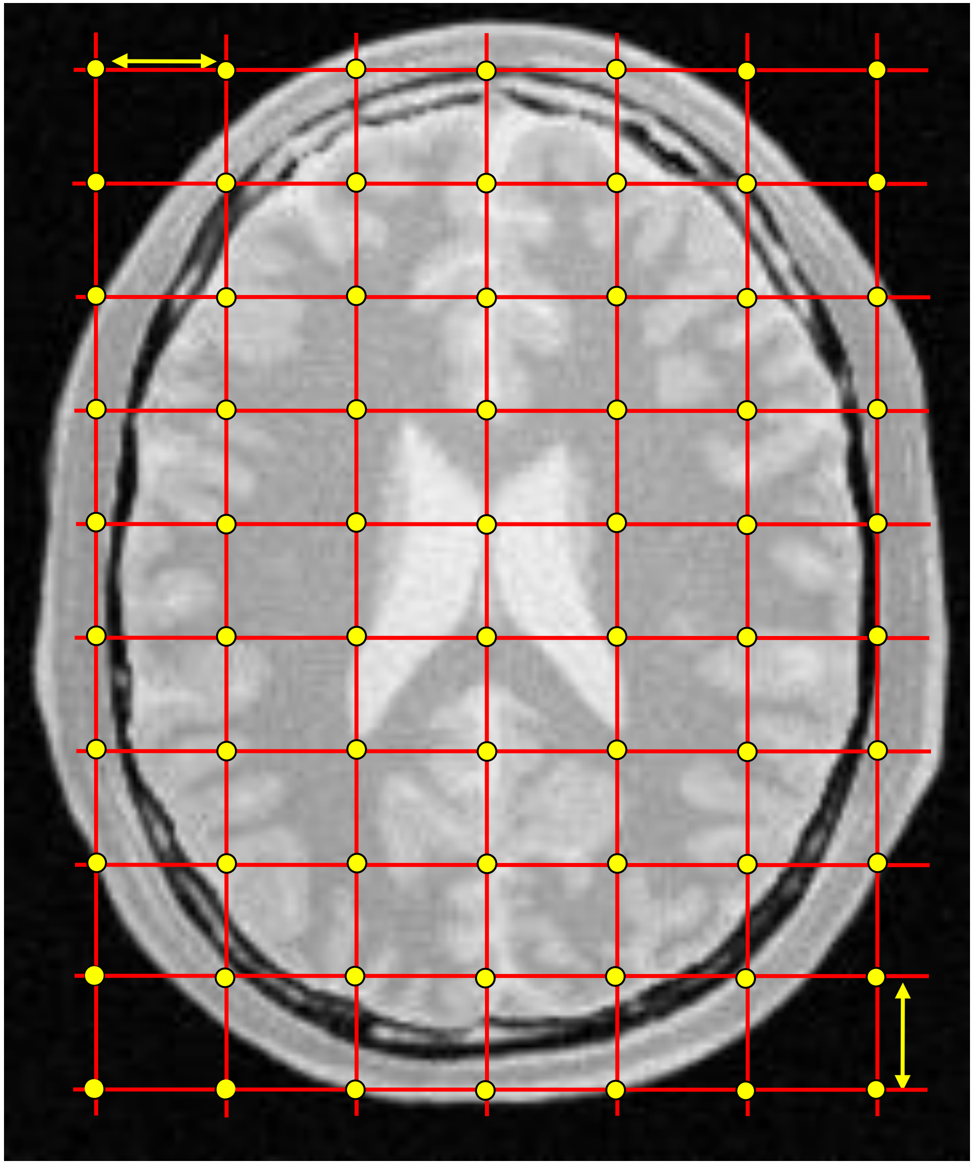
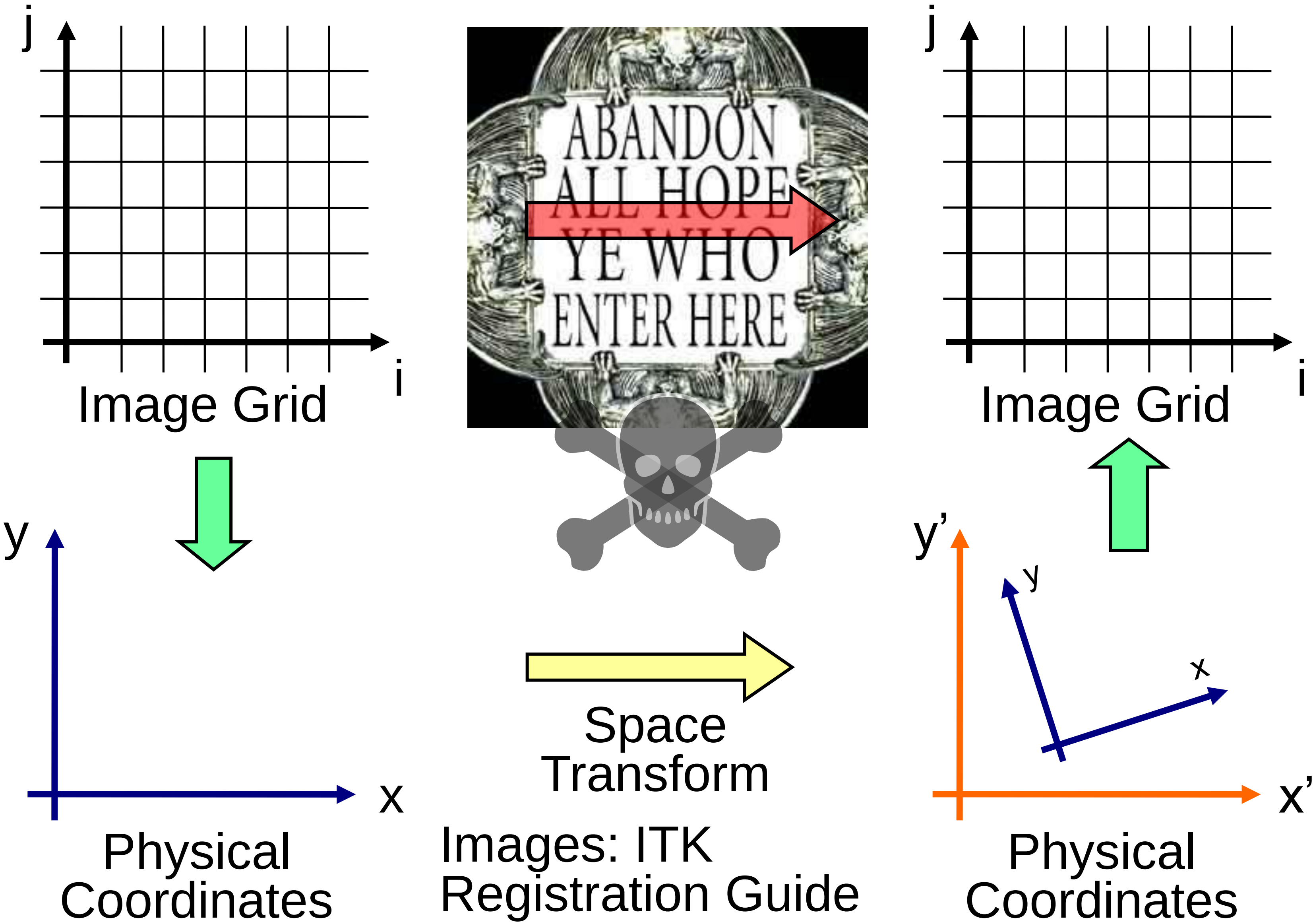


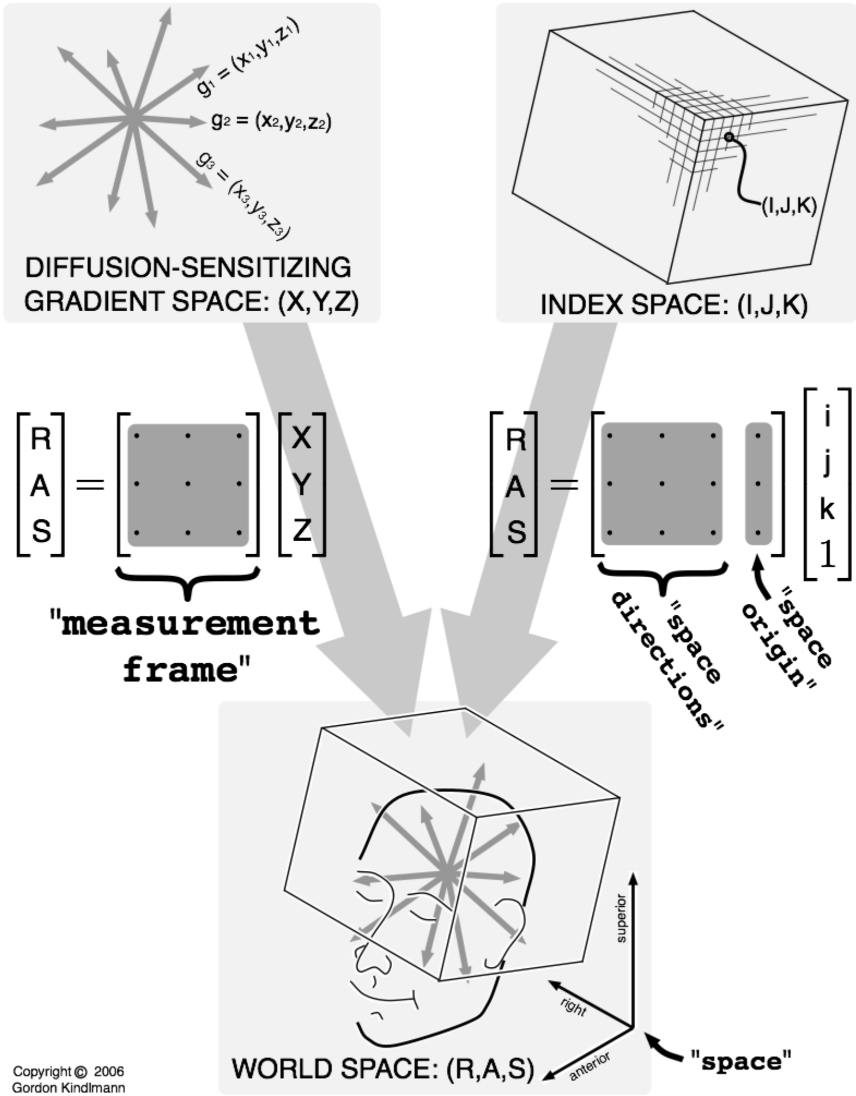
Image: ITK Registration Guide

Coordinate System Conversions



Images: ITK Registration Guide

Coordinate Systems, Example

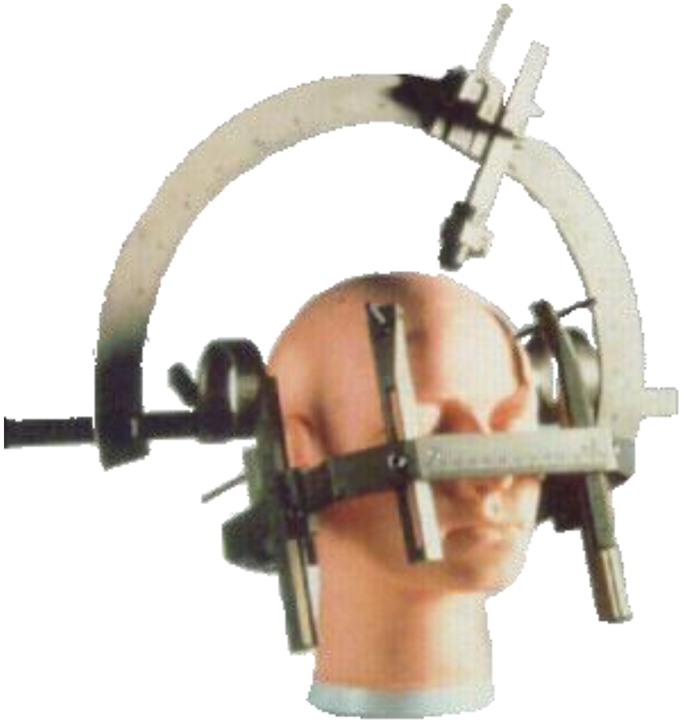


NRRD0005
Complete NRRD file format specification at:
http://teem.sourceforge.net/nrrd/format.html
type: unsigned short
dimension: 4
space: right-anterior-superior
sizes: 256 256 78 59
thicknesses: NaN NaN 2 NaN
space directions: (-0.9375,0,0) (0,-0.9375,0) (0,0,1.7) none
centerings: cell cell cell ???
kinds: space space space list
endian: little
encoding: gzip
space units: "mm" "mm" "mm"
space origin: (-136.14,-130.248,-29.8626)
measurement frame: (1,0,0) (0,1,0) (0,0,1)
data file: caseD00959-dwi-EdCor.raw.gz
modality:=DWMRI
DWMRI_b-value:=700
DWMRI_gradient_0000:= 0.000000 0.000000 0.000000
DWMRI_gradient_0001:= 0.000000 0.000000 0.000000

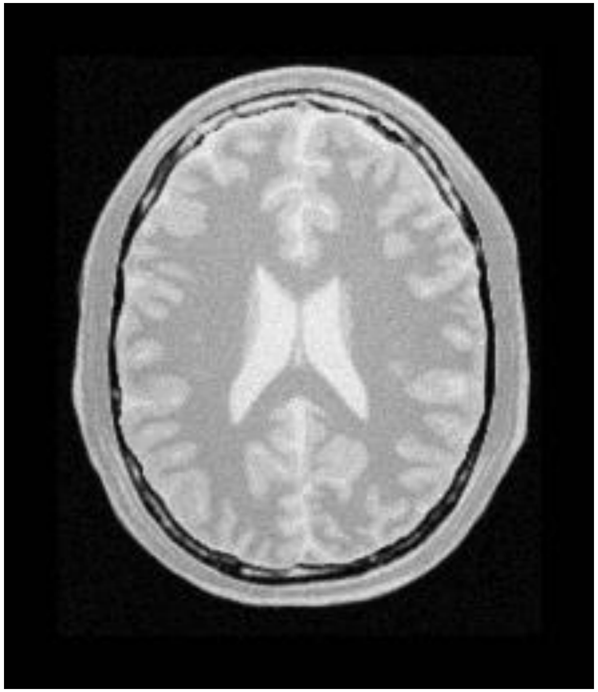
....

Image: Kindlmann

Registering What to What?



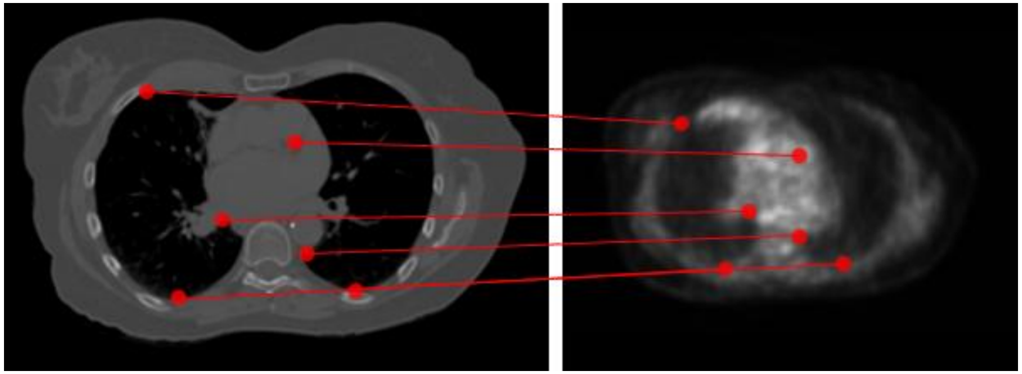
Extrinsic



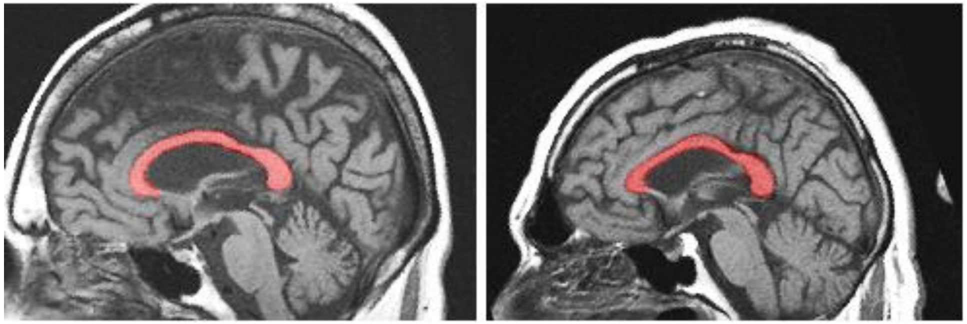
Intrinsic

Registration features derived from image itself

Registration Types



Landmark based



Segmentation based

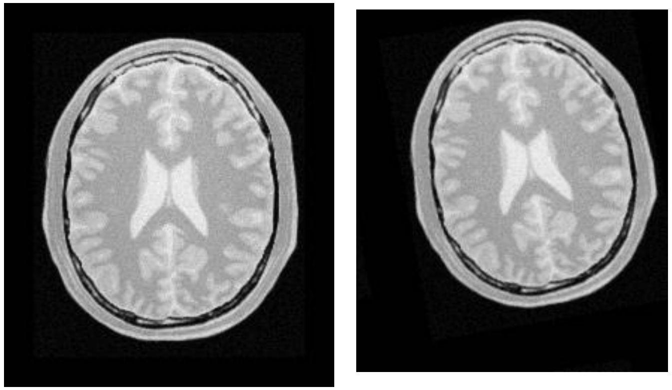
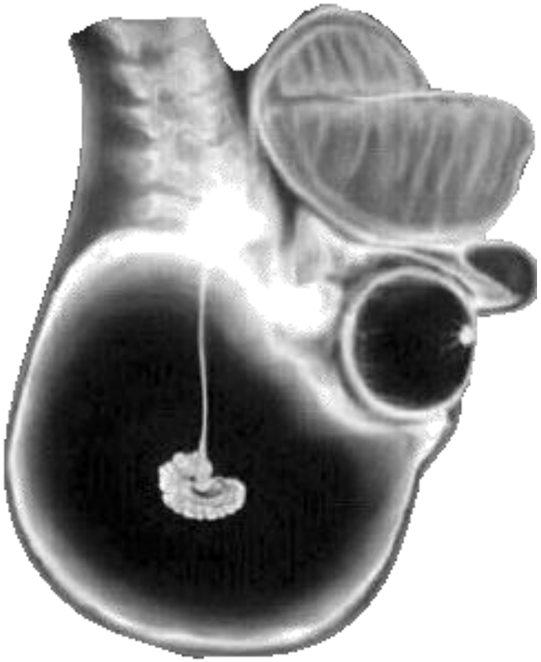
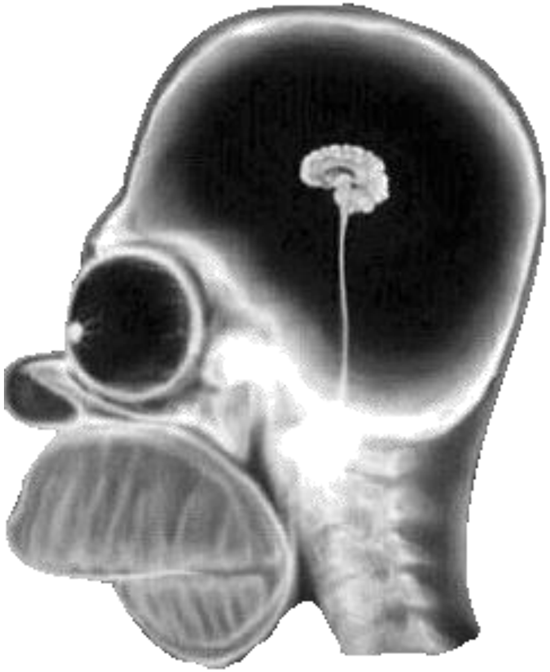


Image based

Images: Alain Pitiot

Registration Types

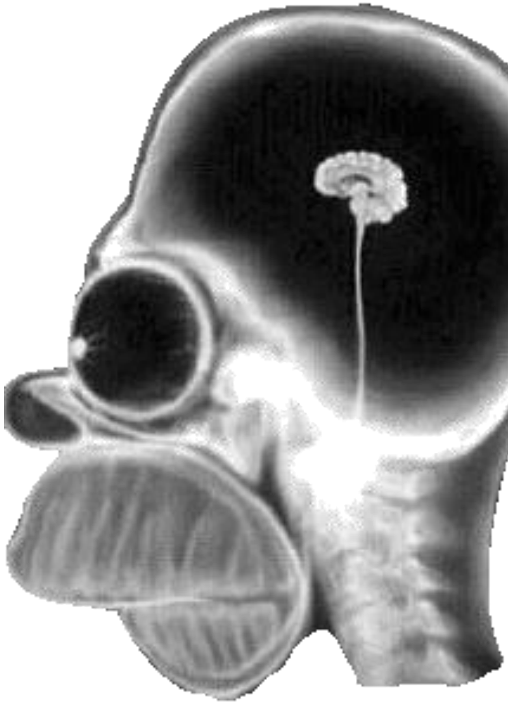


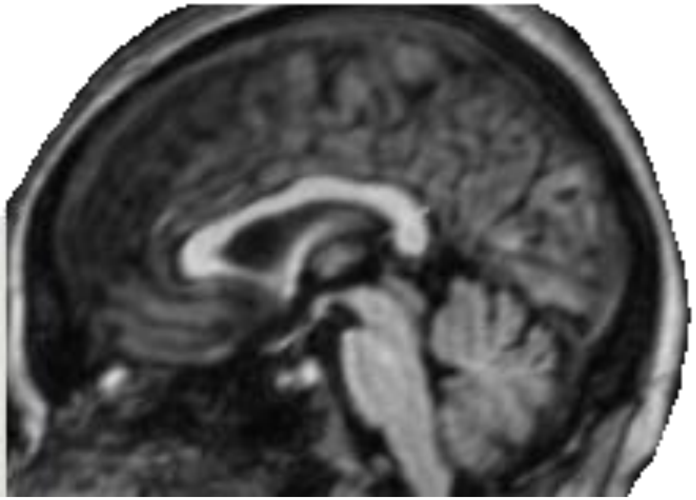


Intra-subject registration
(Within subject)

Images: Alain Pitiot

Registration Types

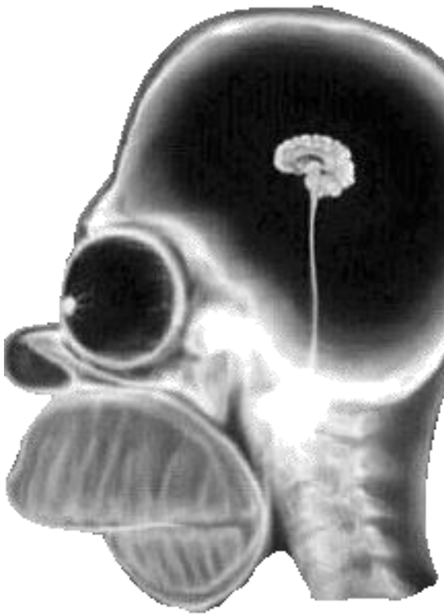


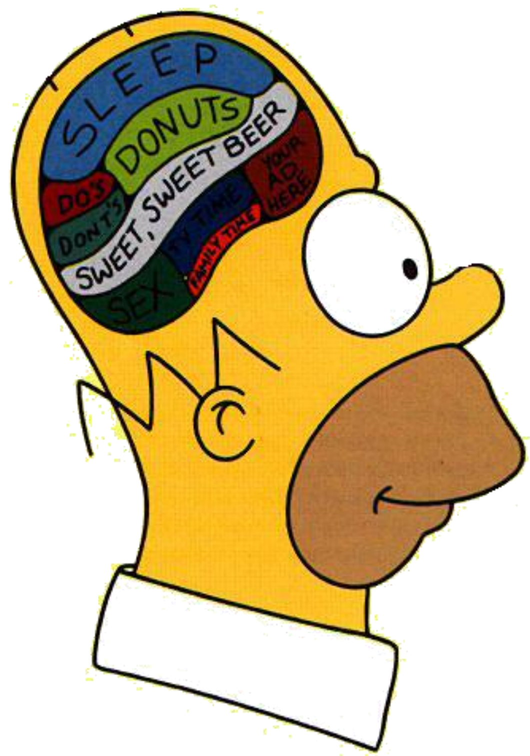


Inter-subject registration
(Between subject)

Images: Alain Pitiot

Registration Types

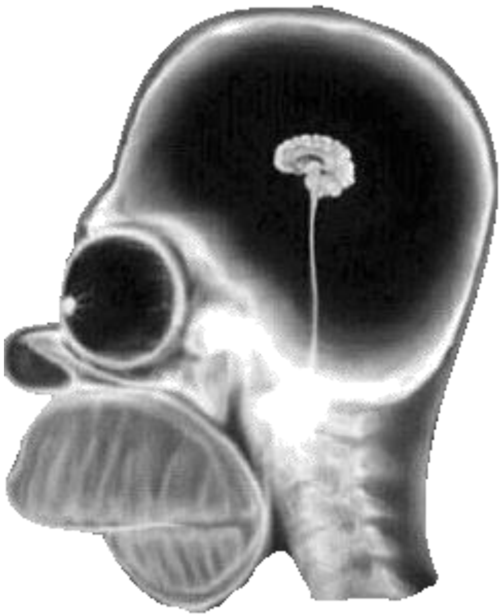


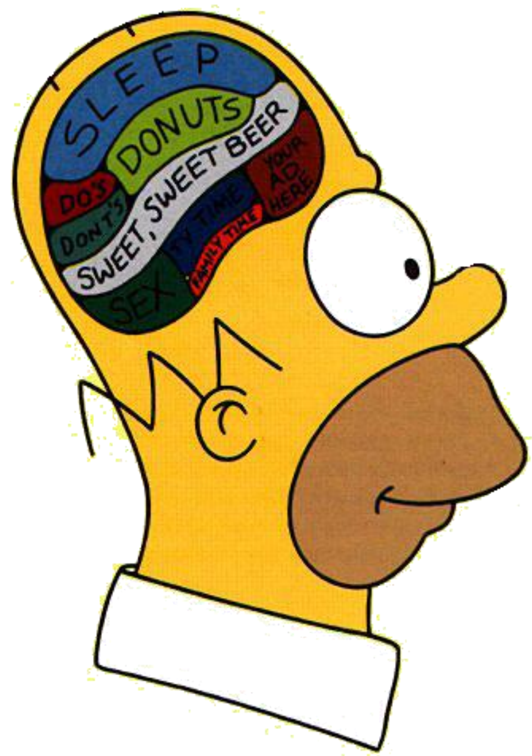


Subject/Atlas Registration

Images: Alain Pitiot

Registration Types



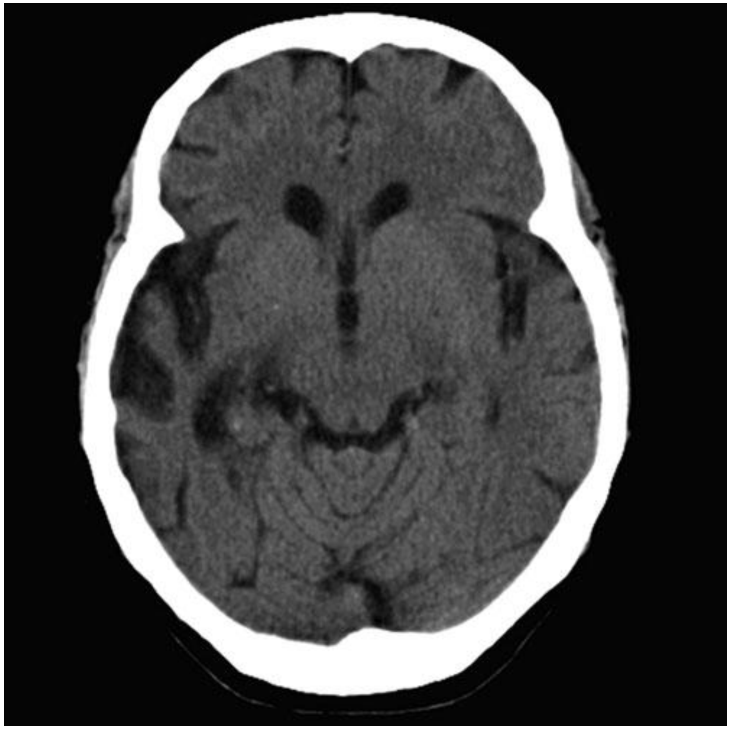


This can be used as a segmentation method
or to add spatial prior information to a segmentation method
= Atlas-based segmentation

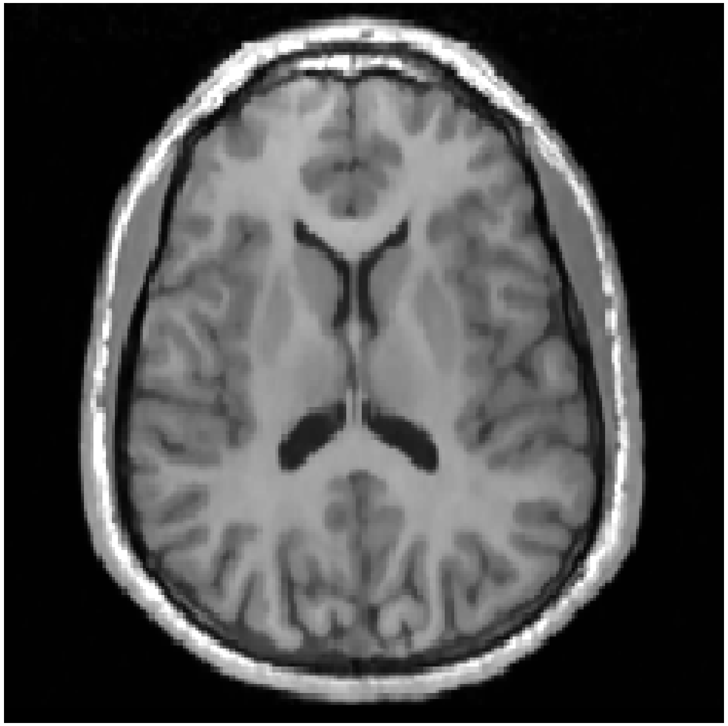
Images: Alain Pitiot

Multi-Modality Registration

X-ray CT



MRI



Requires special care with respect to the similarity metric.
Use for example mutual information.

Image Registration Components

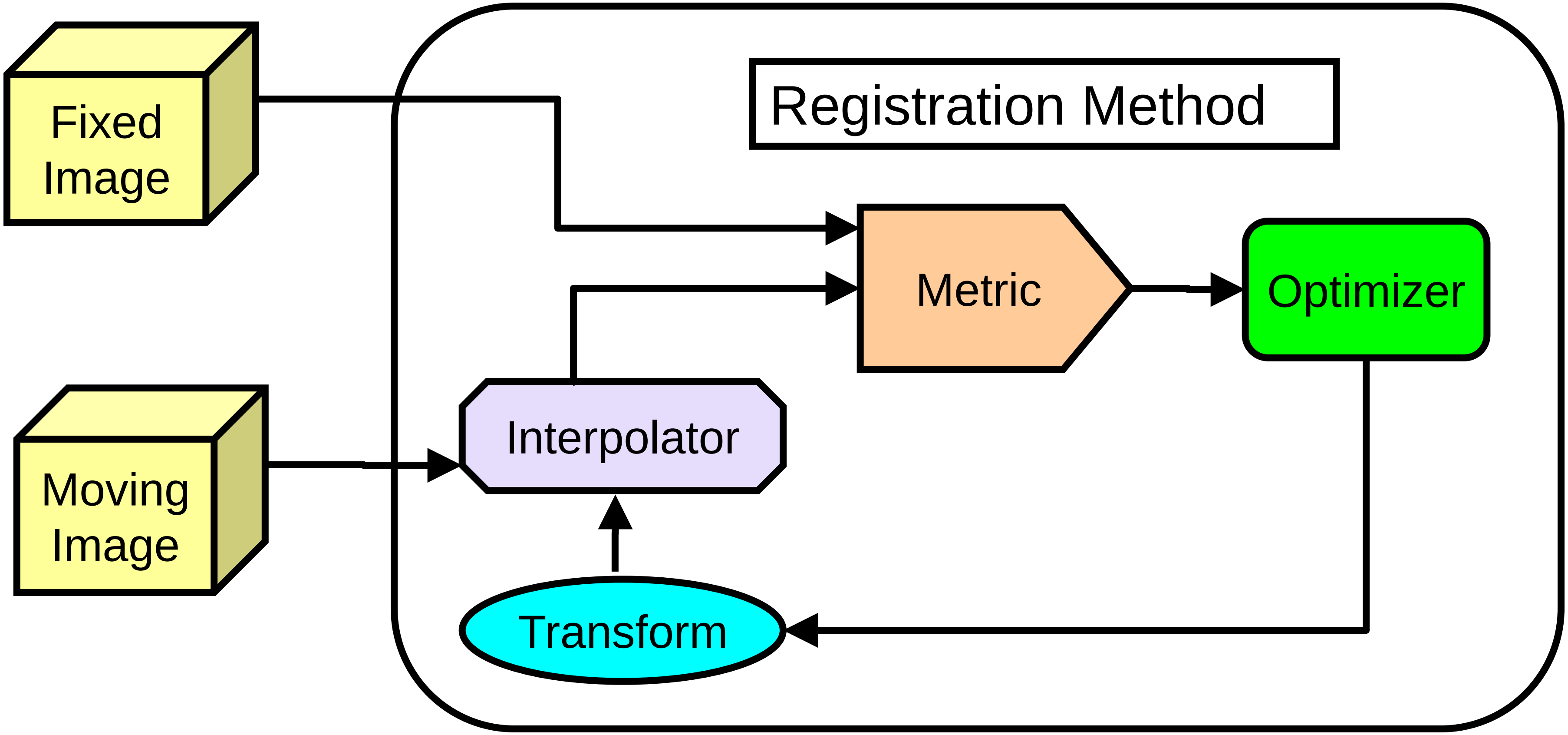


Image: ITK Registration Guide

Interpolation

Determining function values off the grid.

Nearest Neighbor Interpolation:
Good for labelmaps (does not create any new values)

Linear Interpolation

B-Spline Interpolation
Smooth, piecewise bi(tri)-cubic
Achievable by successive subdivision, when
discrete interpolation is what is desired
Used for interpolation (but does not exact match control pts),
but also to represent spatial transformations (see later)

Transformations

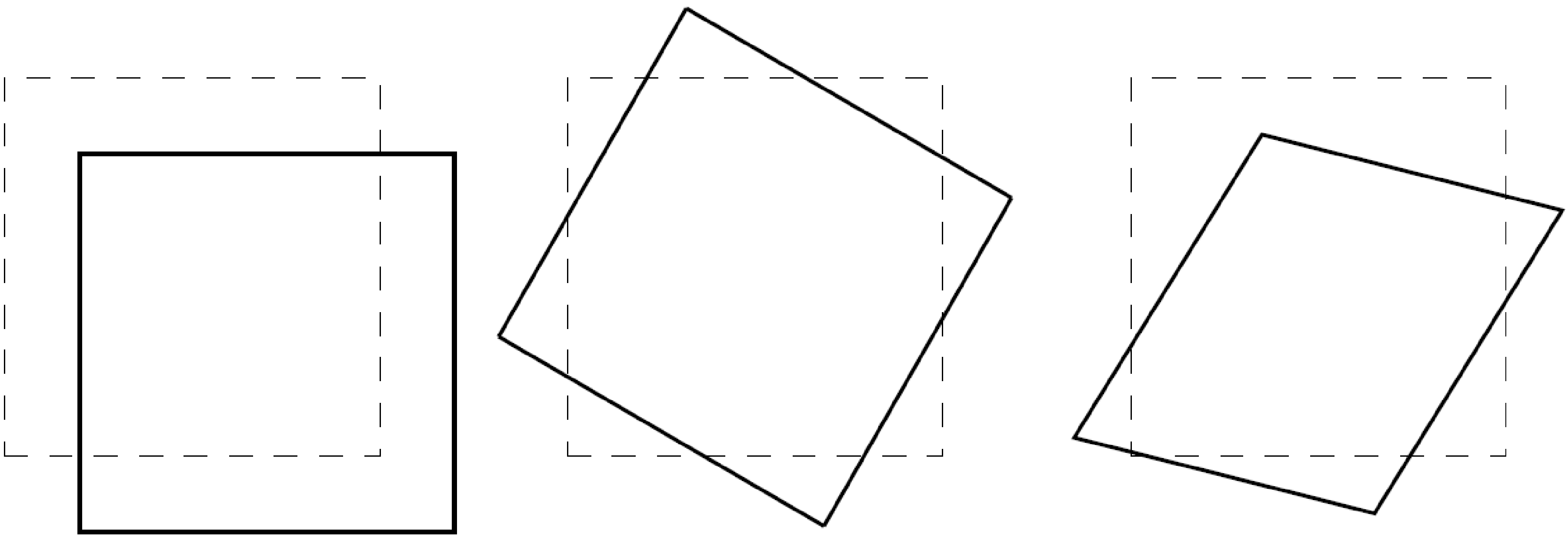
Low-dimensional

Rigid	translation + rotation
Similarity	translation + rotation + scale
Affine	translation + rotation + scale + shear

High-dimensional

Elastic	regularization of displacements (can fold)
Fluid	regularization of velocities (can avoid folding)

Transformations



Translation
$f : \boldsymbol{x} \mapsto \boldsymbol{x} + \boldsymbol{t}$

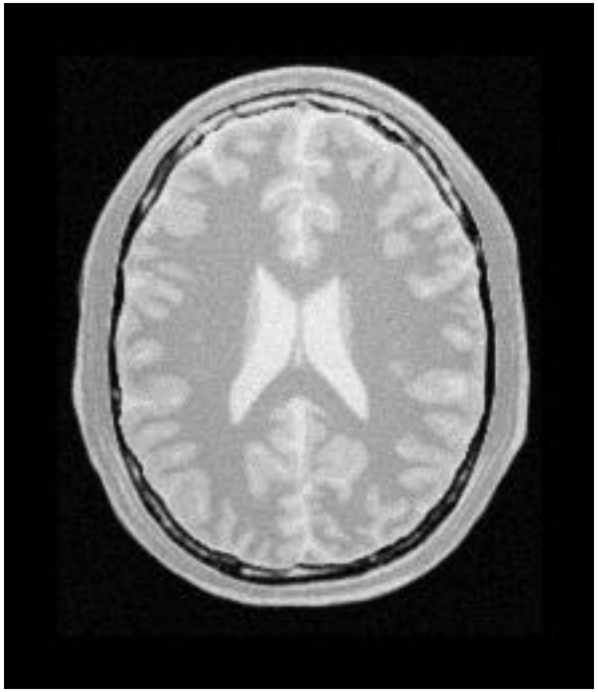
Similarity Transform
$f : \boldsymbol{x} \mapsto sR\boldsymbol{x}$

Affine Transform
$f : \boldsymbol{x} \mapsto A\boldsymbol{x} + \boldsymbol{t}$

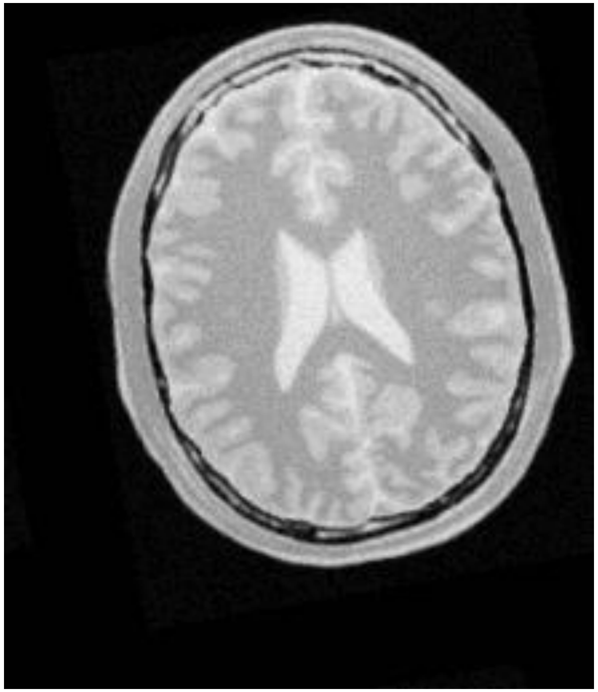
Example affine transformation

$$\underline{x}' = A\underline{x} + \underline{\Delta x}$$

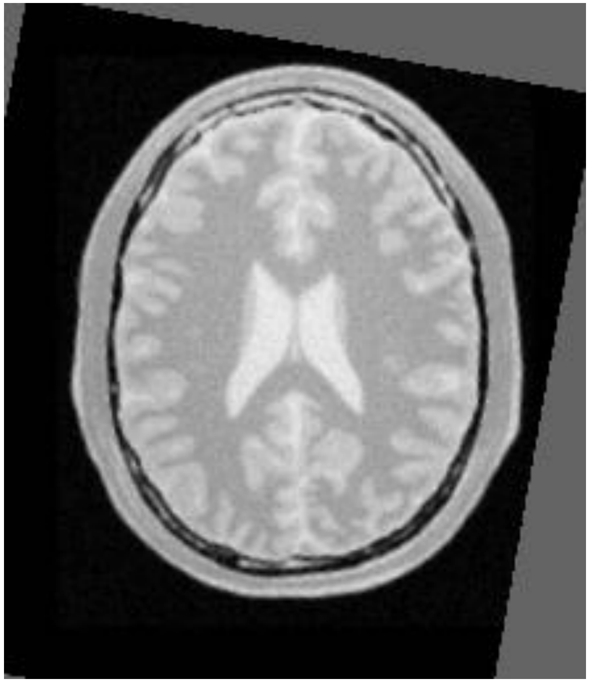
Example Registration



Fixed Image



Moving Image



Registered Moving Image

Images: ITK Registration Guide

Landmark-based registration

Landmarks

Parametric

More parameters = more flexibility

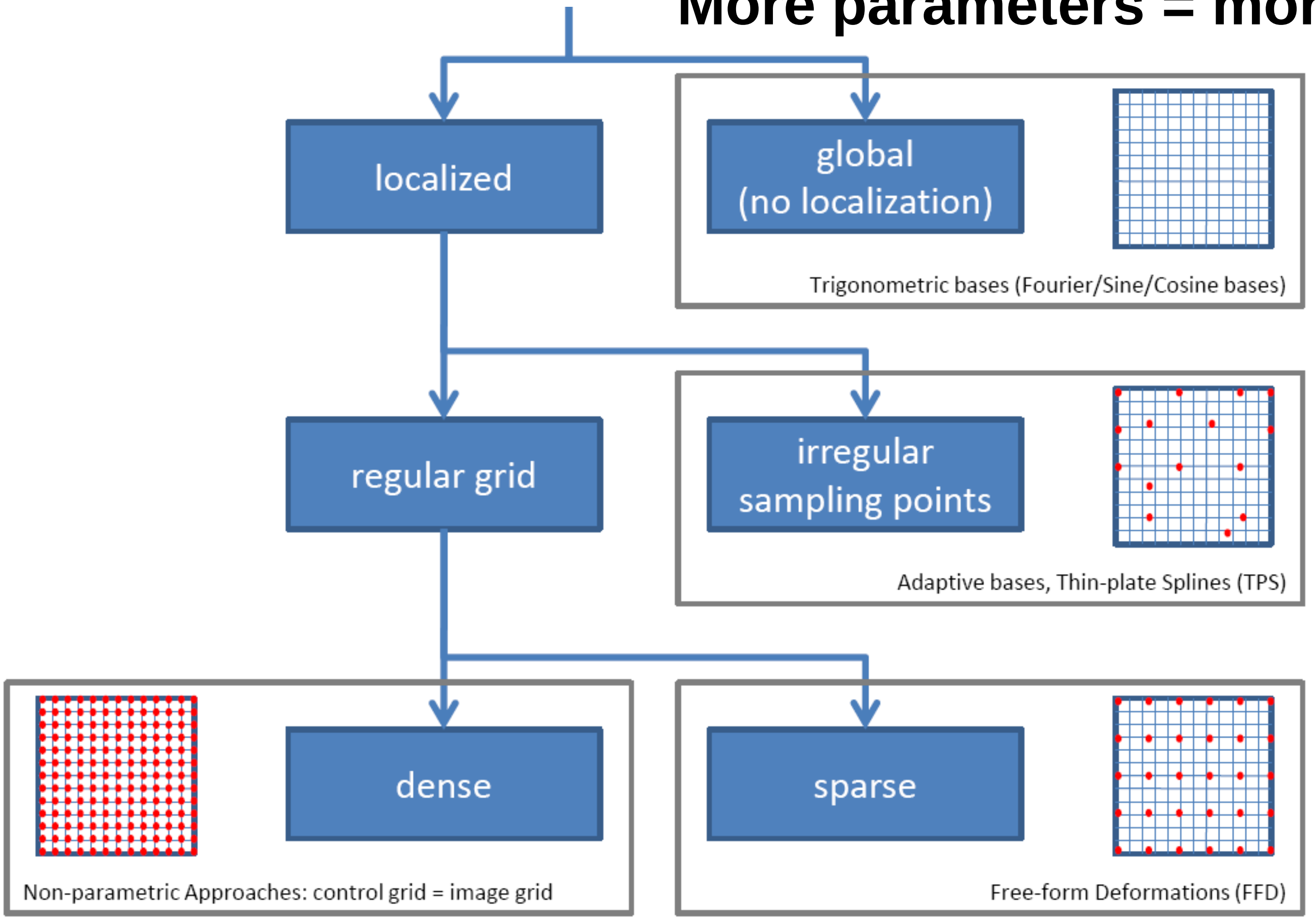


Image: Zikic

Simplest parametric ones are of course rigid, similarity, affine, ...

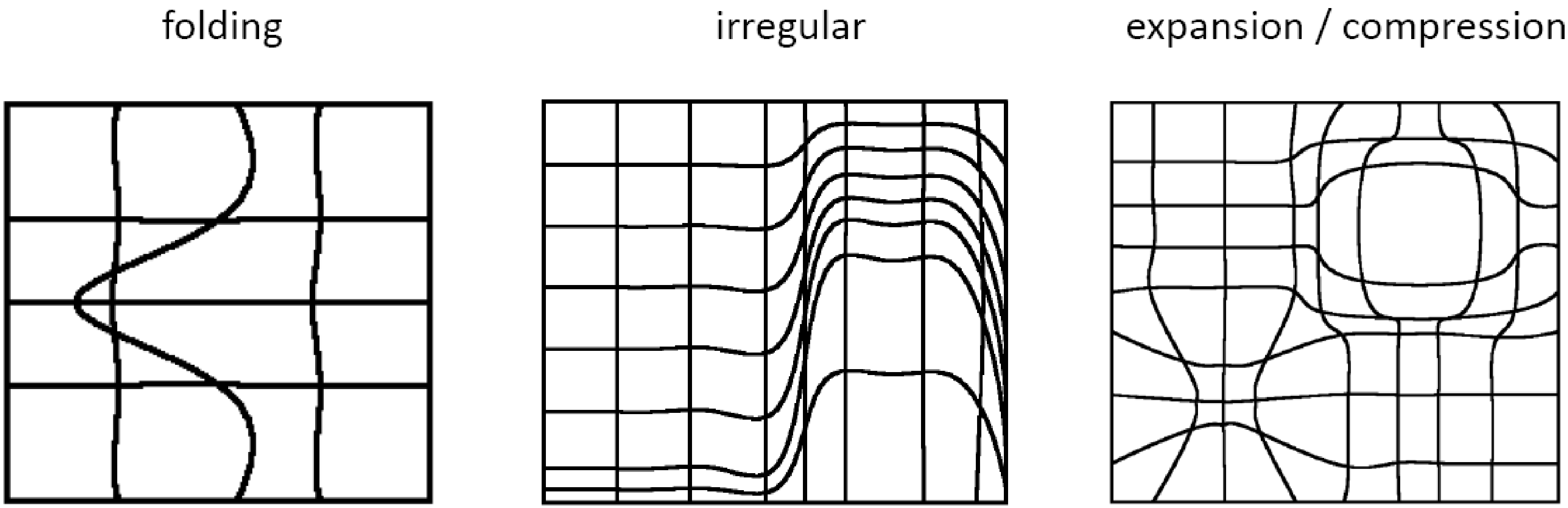
Geometry and Warps Via Landmarks

- Compute $\underline{\Delta x}(\underline{x})$ or a decomposition into translations, rotations, magnifications, & ellipse forming deformations
- Energy options
  - Procrustes energy: for global alignment
  - Thin plate spline bending energy: for exact matching warp, possibly incl. alignment
  - Approximate matching warps:
    - Elastic energy
    - Diffeomorphic flow, e.g., by fluid energy
    - “Freeform” (b-spline) deformations

Geometry and Warps via Landmarks: Issues

- Produces general warp?
- Limited to non-folding warps?
- Energy captures all aspects of warp?
- Symmetric re static vs. moving images?

Want to avoid implausible transformations

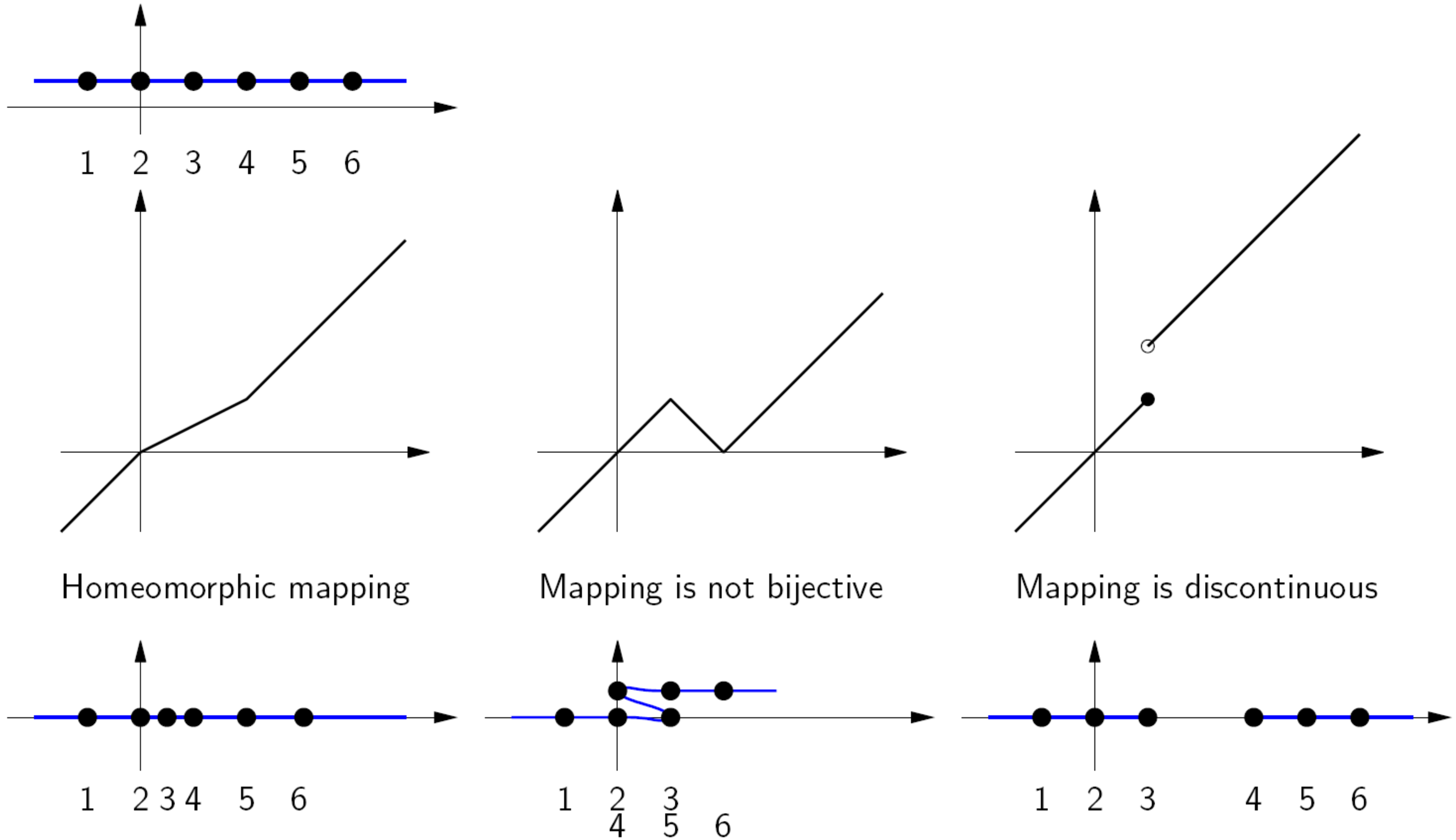


For example, diffeomorphic transformations:

“Bijective, smooth, with a smooth inverse”

Image: Staring

Diffeomorphisms



Is this all that is needed? Are we missing something?

Diffeomorphisms

It seems like a homeomorphism is all we want. Because it

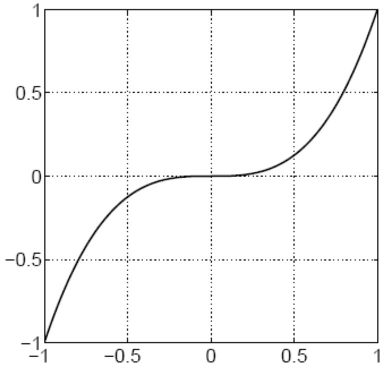
- prevents folding and
- does not allow for tearing either.

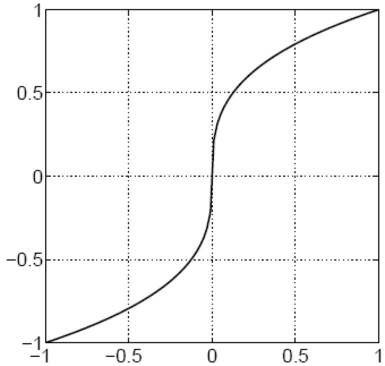
What else could we ask for?

The function

$$f : x \mapsto x^3, \quad f^{-1} : x \mapsto x^{\frac{1}{3}}, \quad x \in \mathbb{R},$$

is homeomorphic. But the derivative of f^{-1} is not defined at 0.





A diffeomorphism is a smooth bijective mapping with a smooth inverse.

Geometric typicality metrics: PDM: Procrustes

- Align shape before warp transformation
  - translation, e.g., to center of mass
  - scaling(?): $|\underline{\mathbf{x}}| = 1$
  - rotation to minimize $|\underline{\mathbf{x}}-\underline{\mathbf{x}}_{std}|$
- Metric = $\Sigma_i \, |x^i - x_{std}{}^i|^2$
  - Has statistical variant
- O(n), with n =number of points
- Symmetric

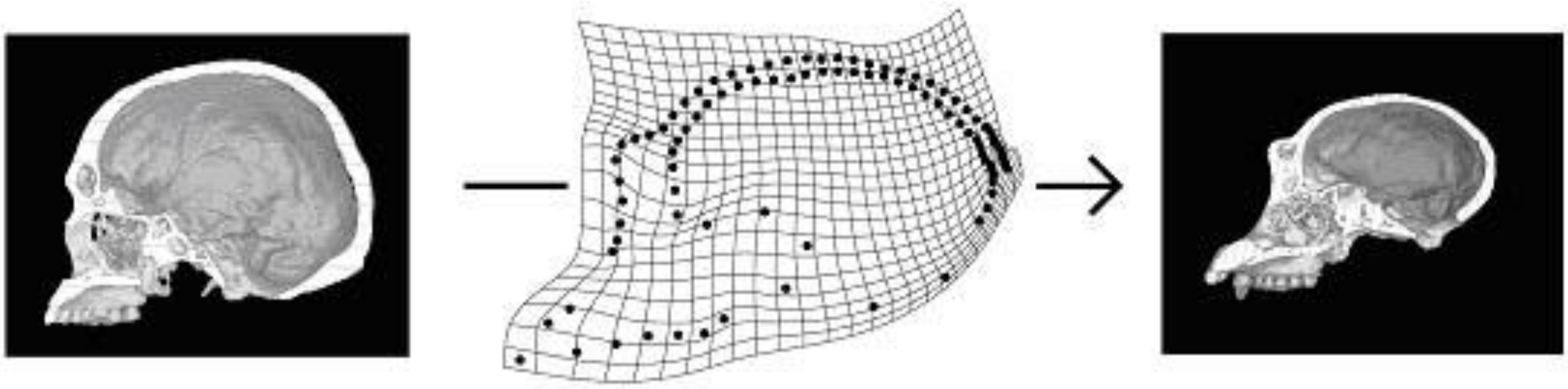
Thin plate spline deformation energy

- Elastic warp in each variable
- Based on landmark primitives
- Minimizing integrated 2nd derivs^2
  - So smooth
- Not necessarily diffeomorphic; may produce folding
- Not symmetric
- Due to Bookstein: Ref: [Dryden & Mardia, *Statistical Shape Analysis*]
- $O(n^2)$

Splines w/ Landmarks

Image: Younes

Thin-plate Splines



Images: University of Vienna

Warping a human skull into a chimpanzee skull.

Thin plate spline deformation energy

Method of landmark matching based on finding the mapping fcn.

$$f(x, y) = a_0 + a_x x + a_y y + \sum_{i=1}^{n} w_i U(|p_i - (x, y)^T|)$$

where $U(r) = -r^2 \log(r^2)$

that matches landmark points to each other while minimizing

$$I_f = \iint \left(\left(\frac{\partial^2 f}{\partial x^2} \right)^2 + 2 \left(\frac{\partial^2 f}{\partial x \partial y} \right)^2 + \left(\frac{\partial^2 f}{\partial y^2} \right)^2 \right) dx\ dy$$

This mapping is not guaranteed to be diffeomorphic.

B-Spline Transformations “Freeform Deformation” [Rueckert]

- *Grid* of control points
- Value at each control point
  - Can be scalar in general
  - For registration, a displacement vector
    - 3 scalars, each separately interpolated
- Patchwise bi(tri)-cubic; smooth at patch boundaries
- Raw form can fold; there is diffeomorphic variant
  - Successive small freeform changes

Diffeomorphic Landmark Matching [Yoshi]

Flowing images into each other. Mapping function $h(\mathbf{x}) = \phi(\mathbf{x}, 1)$ given through the ODE

$$\frac{d\phi(\mathbf{x}, t)}{dt} = v(\phi(\mathbf{x}, t)), \quad t \in [0, 1], \quad \phi(\mathbf{x}, 0) = \mathbf{x}.$$

Minimize smoothness cost subj. to landmark constraints $(h(\mathbf{x}_n) = \mathbf{y}_n)$

$$\hat{v}(\cdot) = \underset{v(\cdot)}{\operatorname{argmin}} \int_0^1 \int_\Omega \|Lv(\mathbf{x}, t)\|^2 \, d\mathbf{x} \, dt.$$

This is guaranteed to give a diffeomorphic h for suitable L (for example $L = I(-\nabla^2 + c)$ works).

Diffeomorphic Landmark Matching

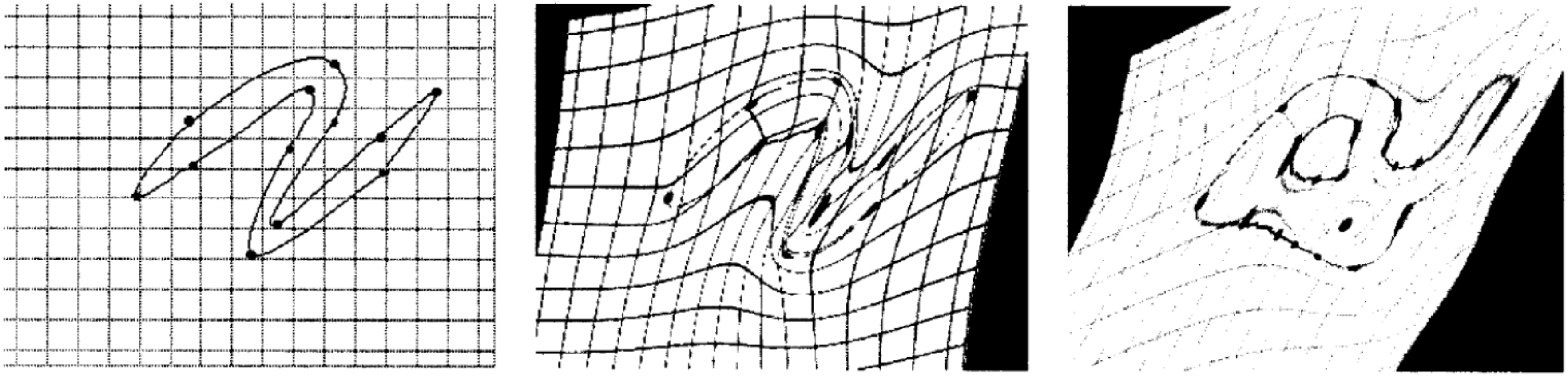


Image from Joshi.

Left: target image, middle: diff. landmark matching, right: small displacement matching.

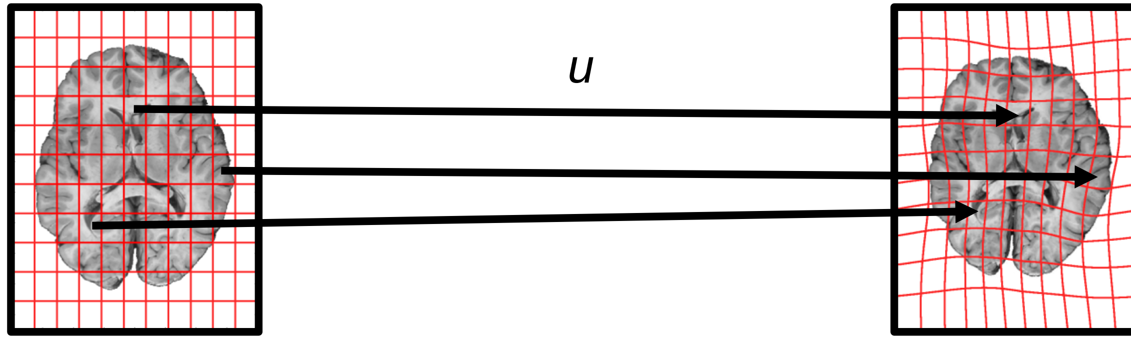
Image-based registration

Image-based registration

Elastic-type versus fluid-type registration

Elastic Registration

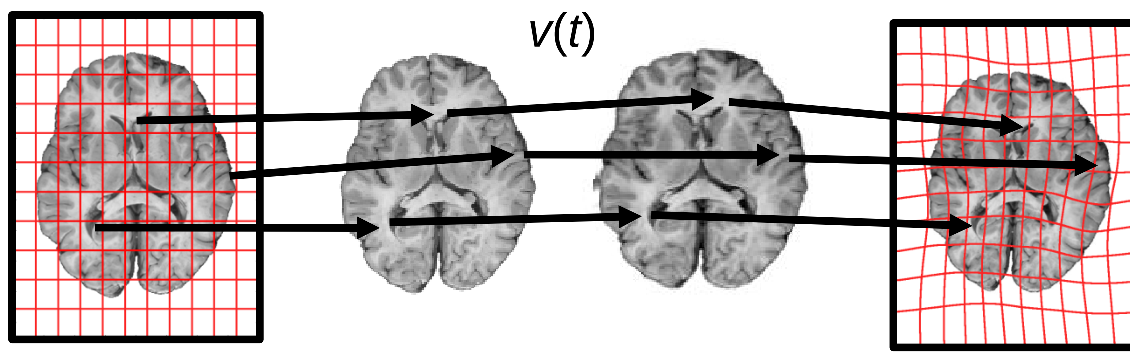
Regularization on the displacement field (*u*)



$$u \quad = \quad \underset{u}{\operatorname{argmin}} \int \Psi_s^u[u] + \frac{1}{\sigma^2}\Psi_d[u, I_0, I_1]\; d\Omega$$

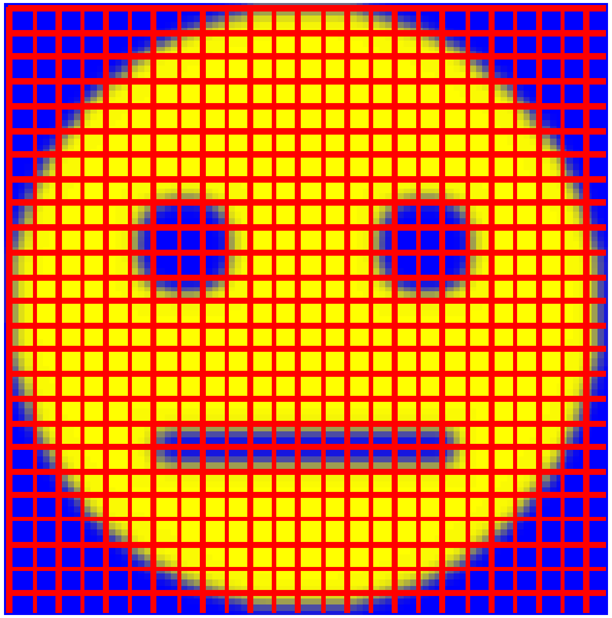
Fluid Registration

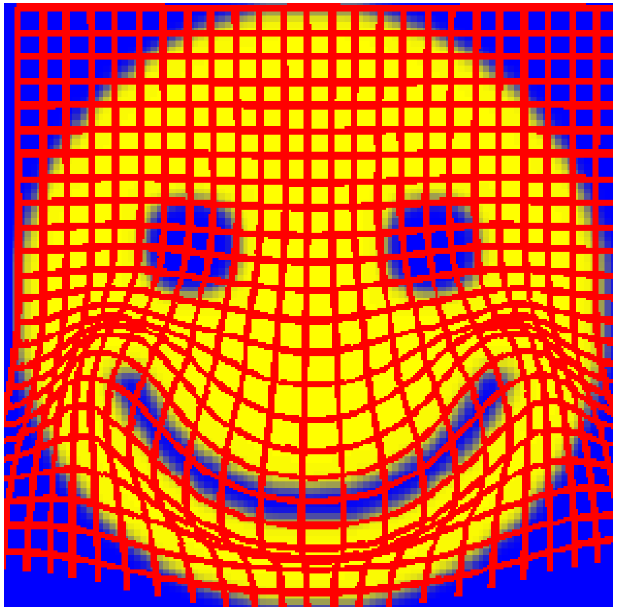
Regularization on the time dependent velocity field (*v*)



$$v \quad = \quad \underset{v}{\operatorname{argmin}} \iint \Psi_s^v[v]\; d\Omega\; dt + \frac{1}{\sigma^2}\int \Psi_d[u, I_0, I_1]\; d\Omega.$$

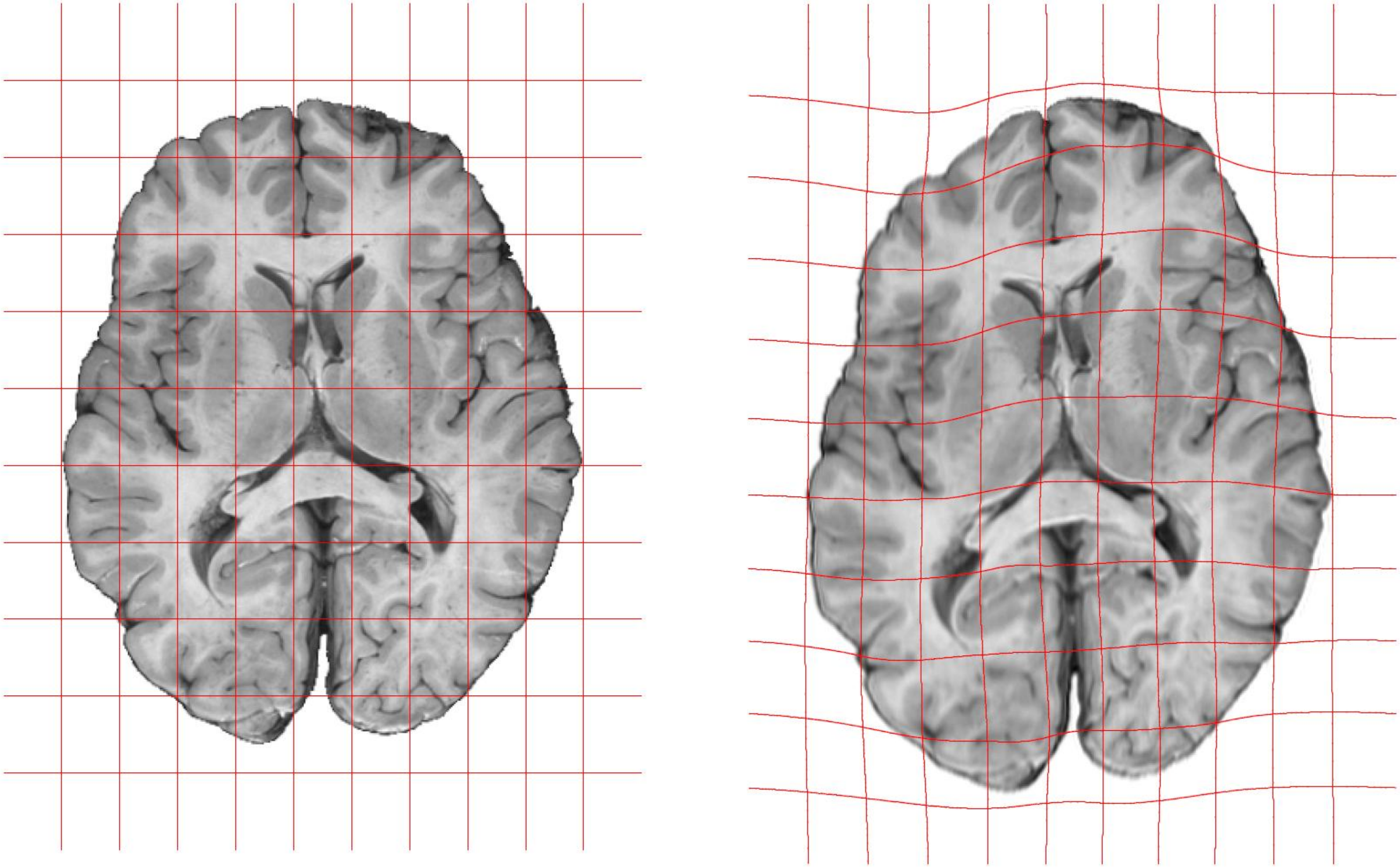
Elastic Transformation





Images: Ashburner

Deformable registration (dense deformation fields)



Many registration methods available.

Displacement-regularized registration

Combination of

- regularization of displacement field u and
- image similarity measure (SSD, correlation, MI, etc.)

$$u^* = \underset{u}{\operatorname{argmin}} \quad \underbrace{S[u]}_{\text{regularization}} \quad + \underbrace{\frac{1}{\sigma^2} D[u, I_0, I_1]}_{\text{similarity measure}}$$

Multiple options for regularization also

- diffusive
- curvature
- elastic
- ...

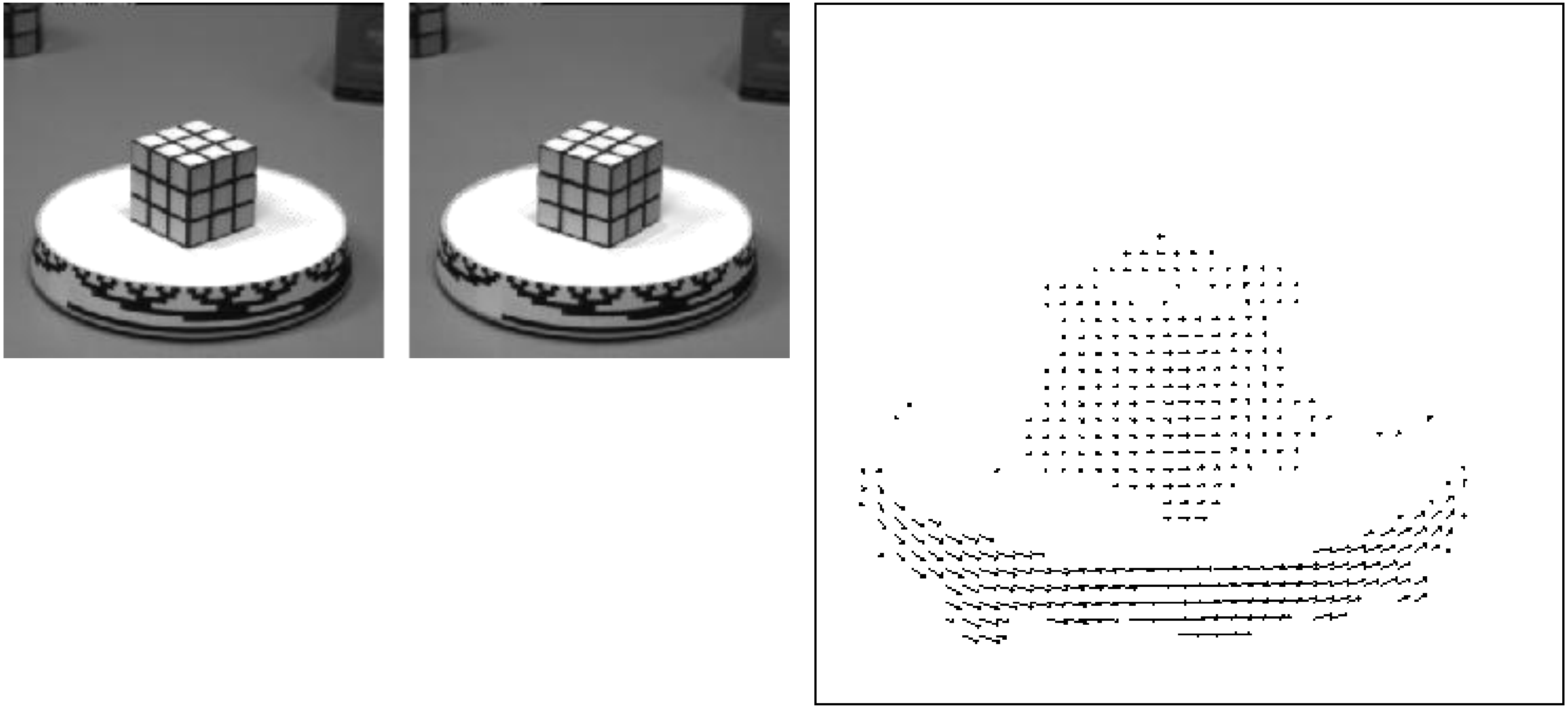
Diffusion Regularization (=Optical Flow)

Enforces smoothness of the displacement fields
(component by component)

$$S[u] = \frac{1}{2}\sum_{l=1}^{d}\int_{\Omega} \|\nabla u_l\|^2 \ dx \overset{2D}{=} \frac{1}{2}\int_{\Omega} \|\nabla u_1\|^2 + \|\nabla u_2\|^2 \ dx$$

Simplest model, as used for example in optical flow
(Gradients will result in Laplacian terms -> smoothing)

Optical Flow



$$E(v^x, v^y) = \int_\Omega (I_t + I_x v^x + I_y v^y)^2 + \alpha(\|\nabla v^x\|^2 + \|\nabla v^y\|^2)\ d\Omega$$

Images: Slides of Bill Freeman

Curvature Regularization

Regularization based on the Laplacian of the displacements

$$S[u] = \frac{1}{2}\sum_{l=1}^{d}\int_{\Omega}(\Delta u_l)^2 \; dx \overset{2D}{=} \frac{1}{2}\int_{\Omega}(\Delta u_1)^2 + (\Delta u_2)^2 \; dx$$

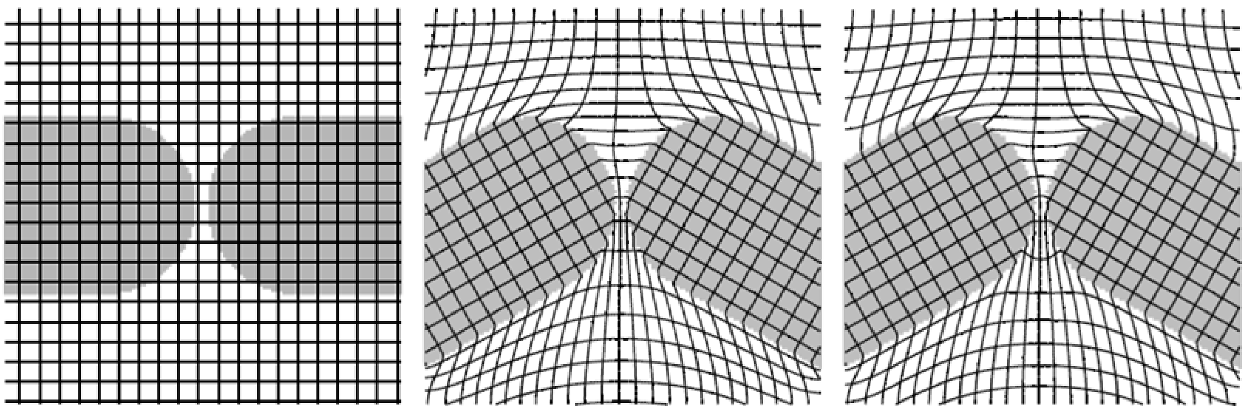
Invariant to affine transformations (due to second derivatives).

Elastic Regularization

Based on physical model of linear elasticity

$$S[u] = \int_\Omega \frac{\mu}{4} \sum_{j,k=1}^{d} \underbrace{(\partial_{x_j} u_k + \partial_{x_k} u_j)^2}_{\text{rigidity}} + \underbrace{\frac{\lambda}{2} (\text{div } u)^2}_{\text{change in material volume}} \, dx = \underbrace{P[u]}_{\text{elastic potential}}$$

μ, λ: Lame constants (control elastic behavior)



Undeformed $\qquad \nu = -1 \qquad \nu = 0$

Elastic Regularization

Elasticity model is the continuum mechanics equivalent to the spring model

$$E[u] = \frac{1}{2}ku^2$$

Can compute the force by differentiation

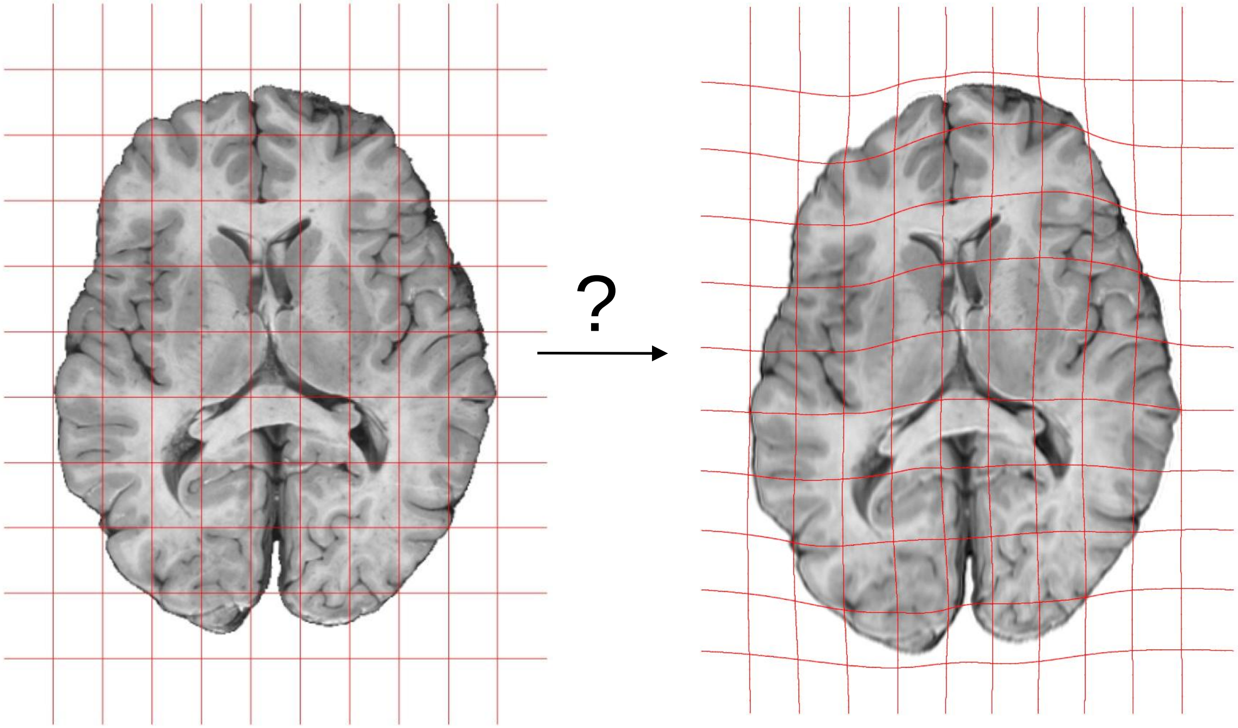
$$F[u] = -\frac{dE(u)}{du} = -ku$$

In the continuum, the force is obtained through the variation

$$f = \mu\Delta u + (\lambda + \mu)\nabla \mathrm{div}(u)$$

which, needs to be balanced w/ force form similarity measure

Fluid flow registration



What is the best velocity field, v, to deform one image into the other?

Fluid flow setup [Miller et al.]:

$$E(v) = \int_0^1 \|v\|_V^2 \; dt + \frac{1}{\sigma^2} \|I_0 \circ \Phi_{1,0} - I_1\|_{L_2}^2$$

Fluid flow registration

Fluid flow setup [Miller et al.]:

$$E(v) = \int_0^1 \|v\|_V^2 \; dt + \frac{1}{\sigma^2}\|I_0 \circ \Phi_{1,0} - I_1\|_{L_2}^2$$

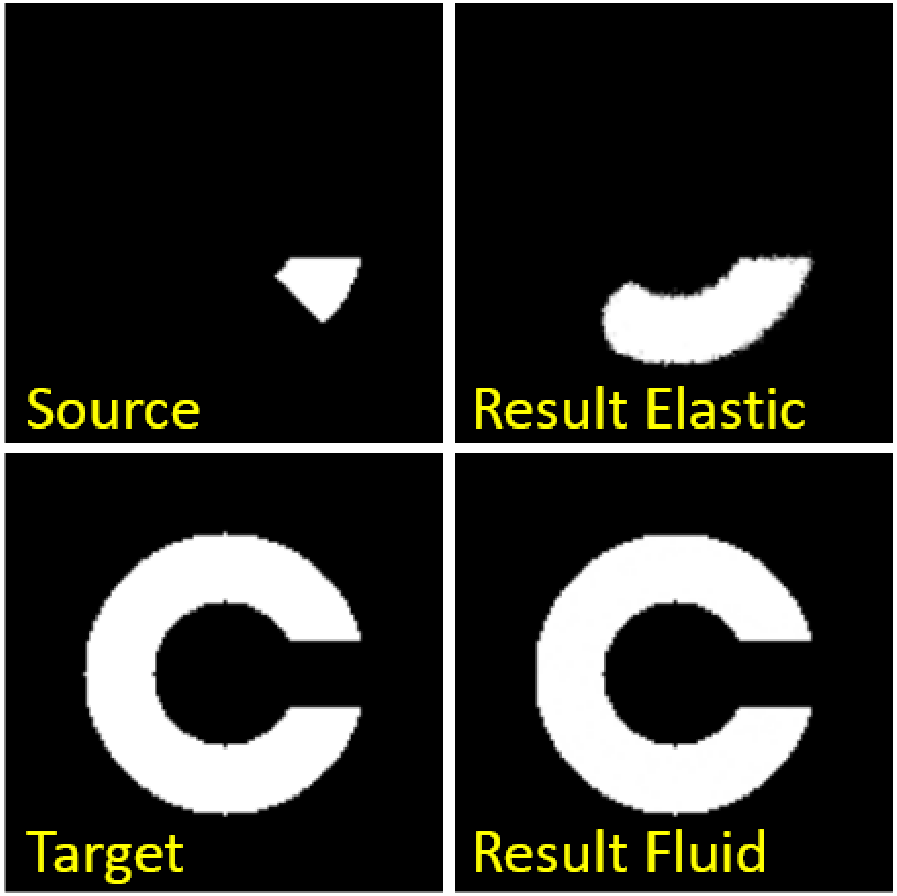
Complex optimization problem:

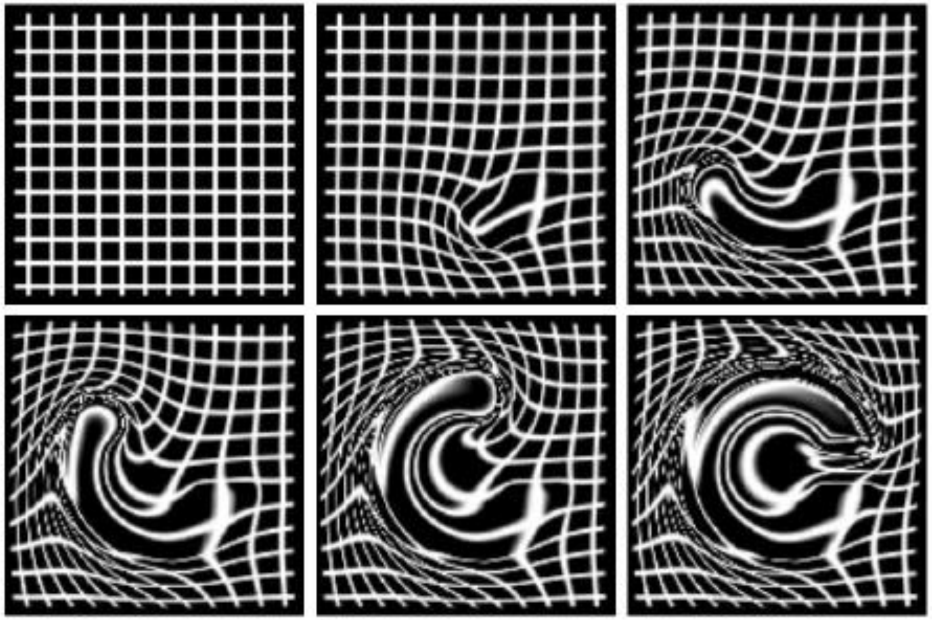
- v depends on time and space
- requires solution of full space-time problem
- often approximately solved using a greedy algorithm (greedy versions of LDDMM, Demons, ...)

Test cases

Test cases

Non-parametric: Elastic- vs. fluid-type registration

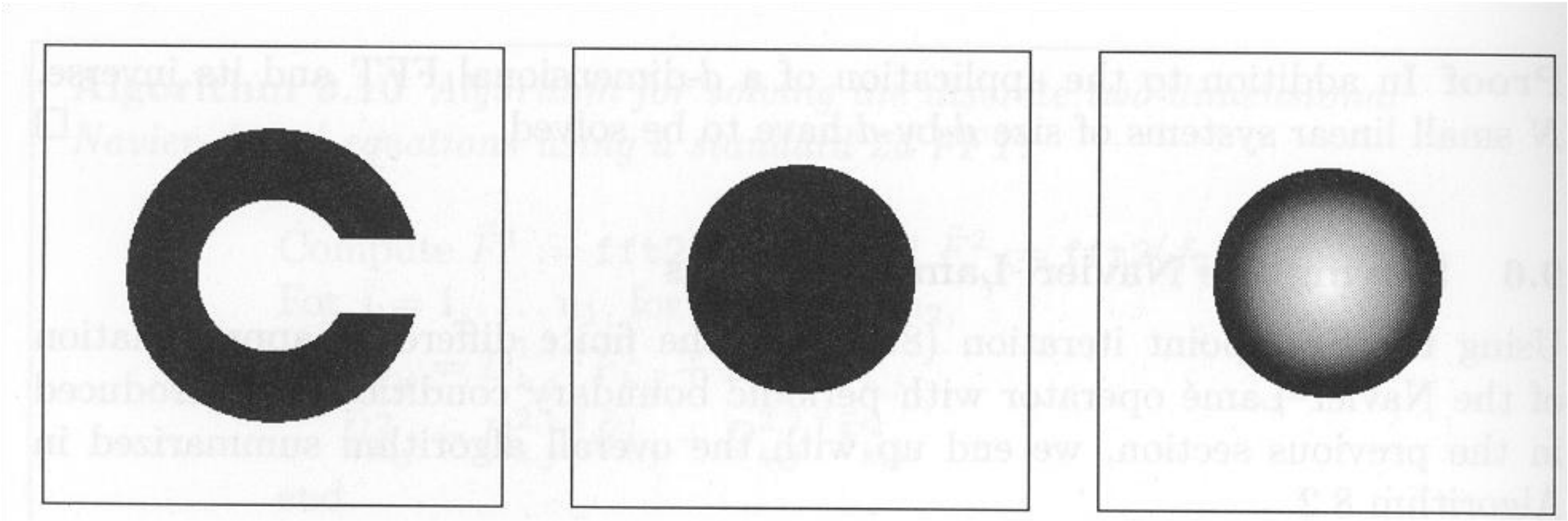




Images: Christensen

Test Cases

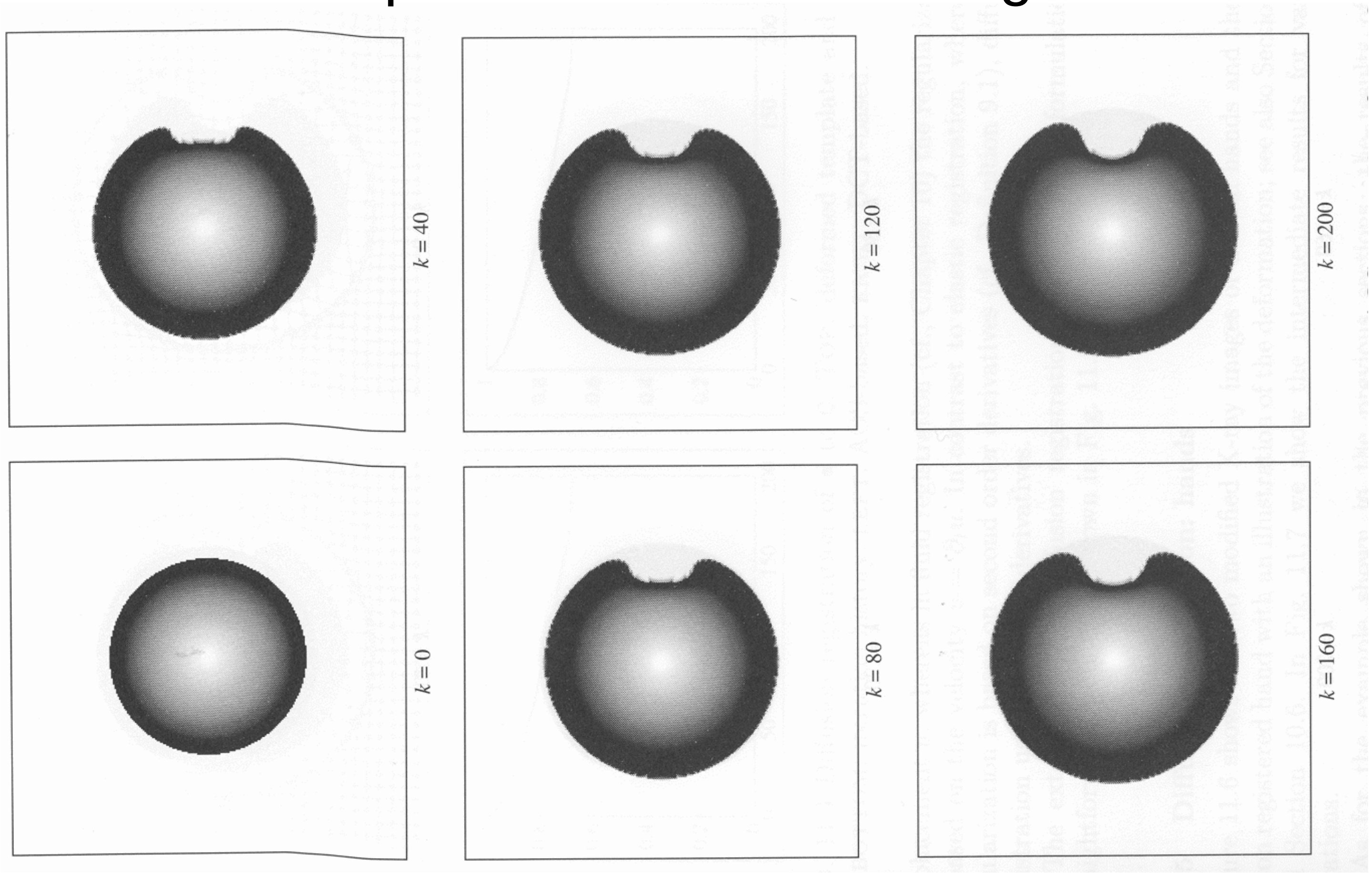
Standard test cases to assess registration behaviour for different registration algorithms



Images: Modersitzki

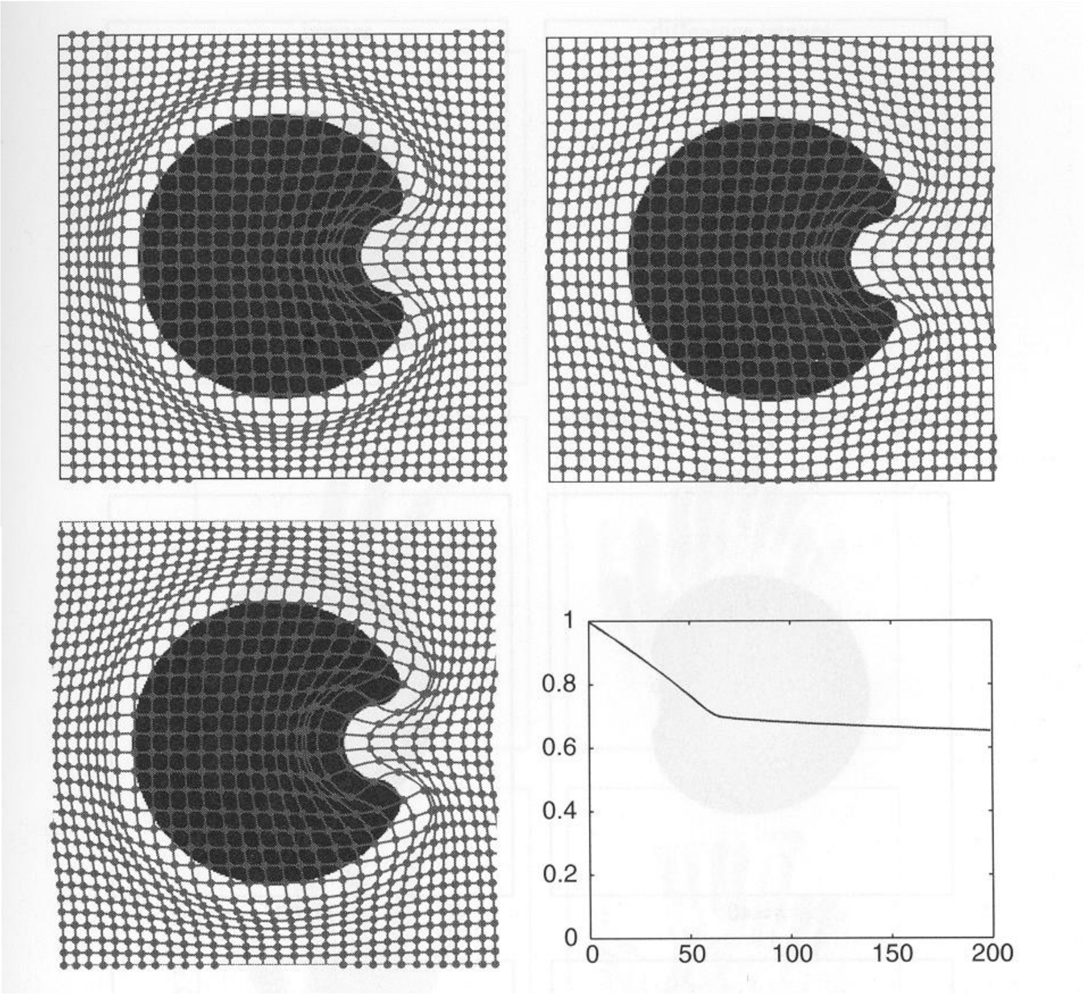
Diffusion Registration

Regularization of displacements hinders large deformations



Images: Modersitzki

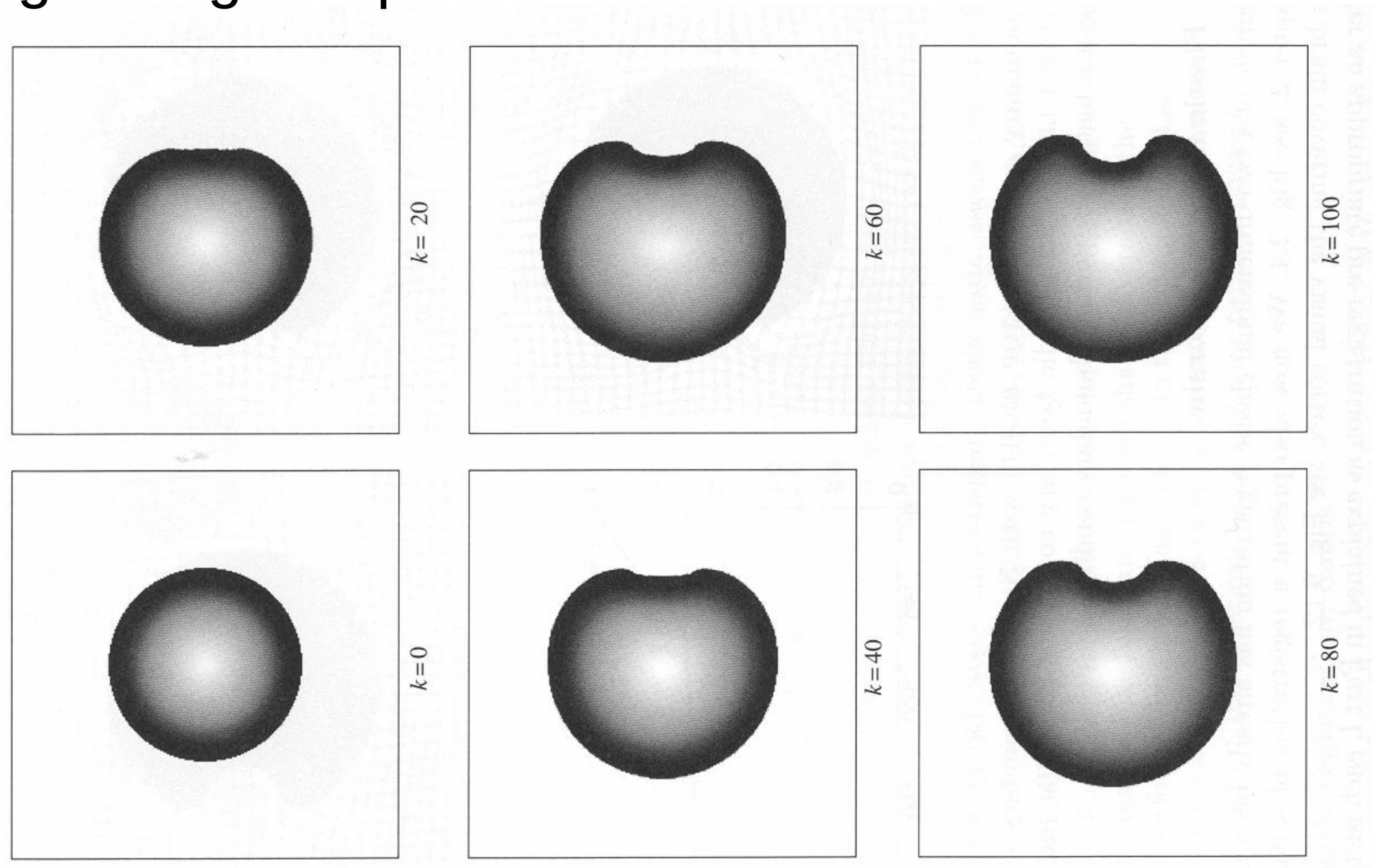
Elastic Registration



Images: Modersitzki

Elastic Registration

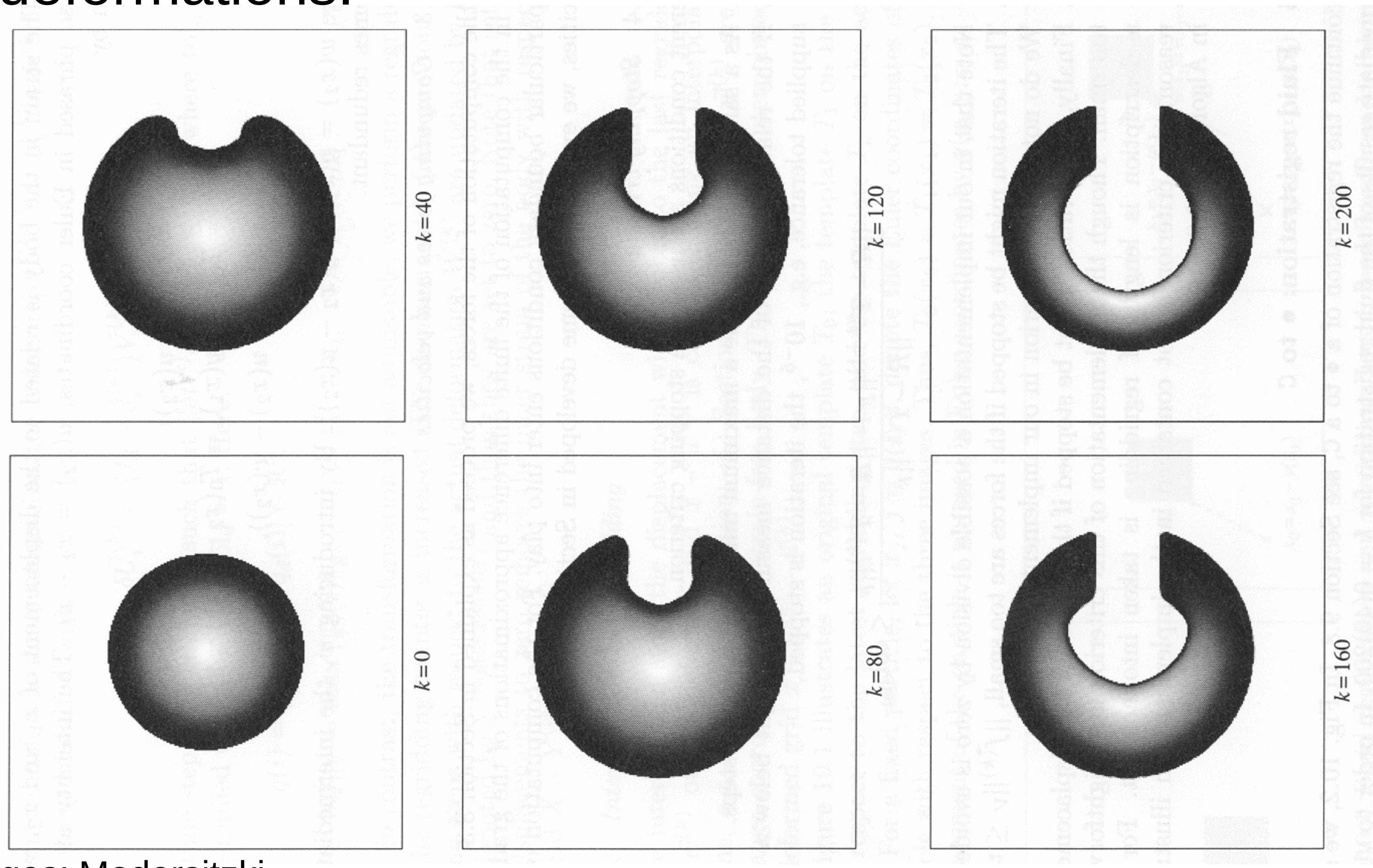
As for diffusive regularization, the regularization of displacements discourages large displacements



Images: Modersitzki

Fluid Registration [Christensen, Joshi, Miller]

Since fluid registrations regularize velocities they allow for large deformations.



Images: Modersitzki

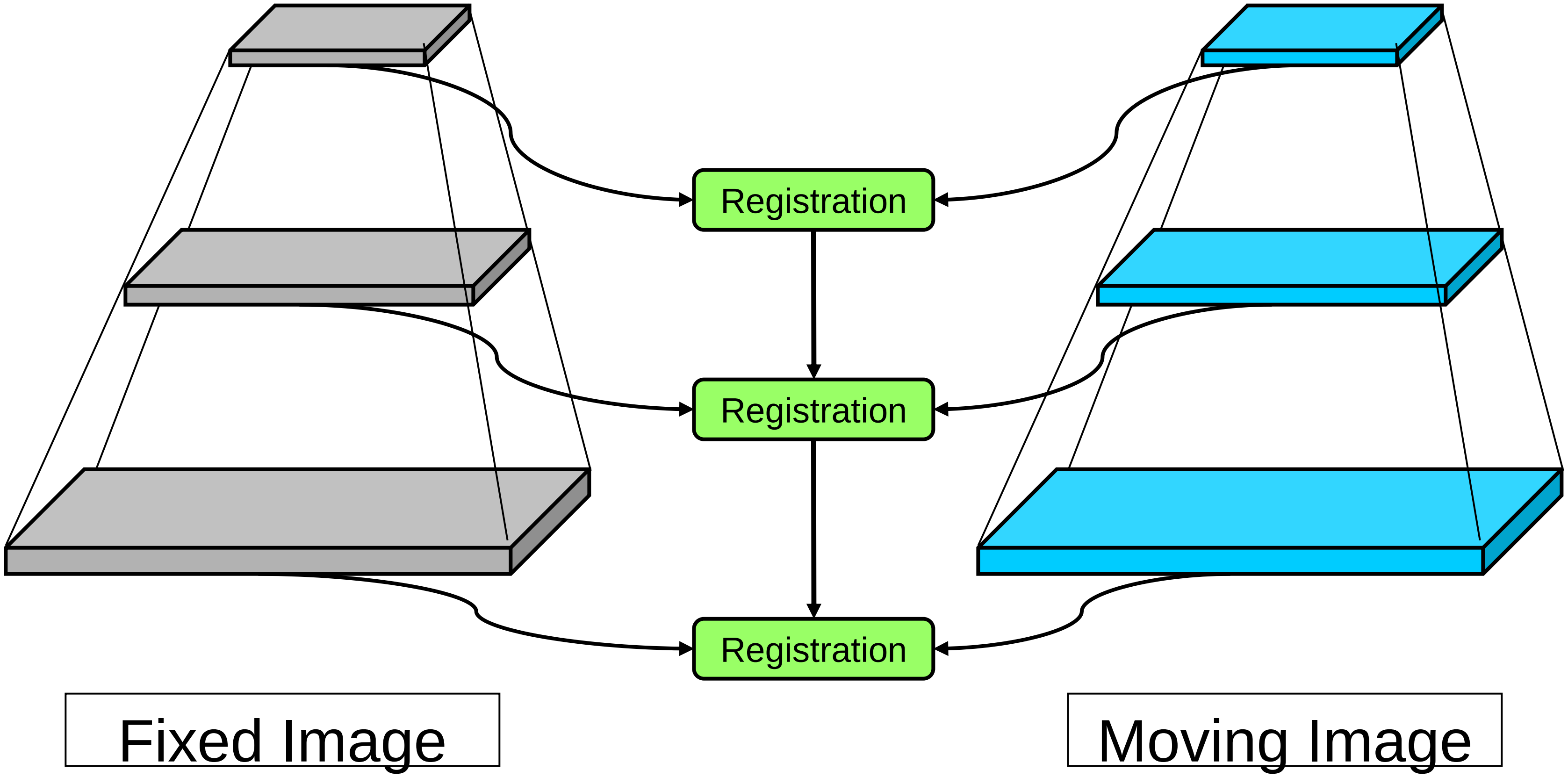
General Issues

General Issues

Warp Transformation on Landmarks or Image Data (or both)

- Typically pre-align
  - Result is different if you don't pre-align
- Objective function terms:
  - Regularization energy
    - Weight value is hard to determine, esp. since two terms are in different units
  - Geometry-to-data match
    - For landmarks
    - For image features
- Does not give exact match due to regularization
- Often will still fall into local energy minima
  - Especially if you do not pre-align

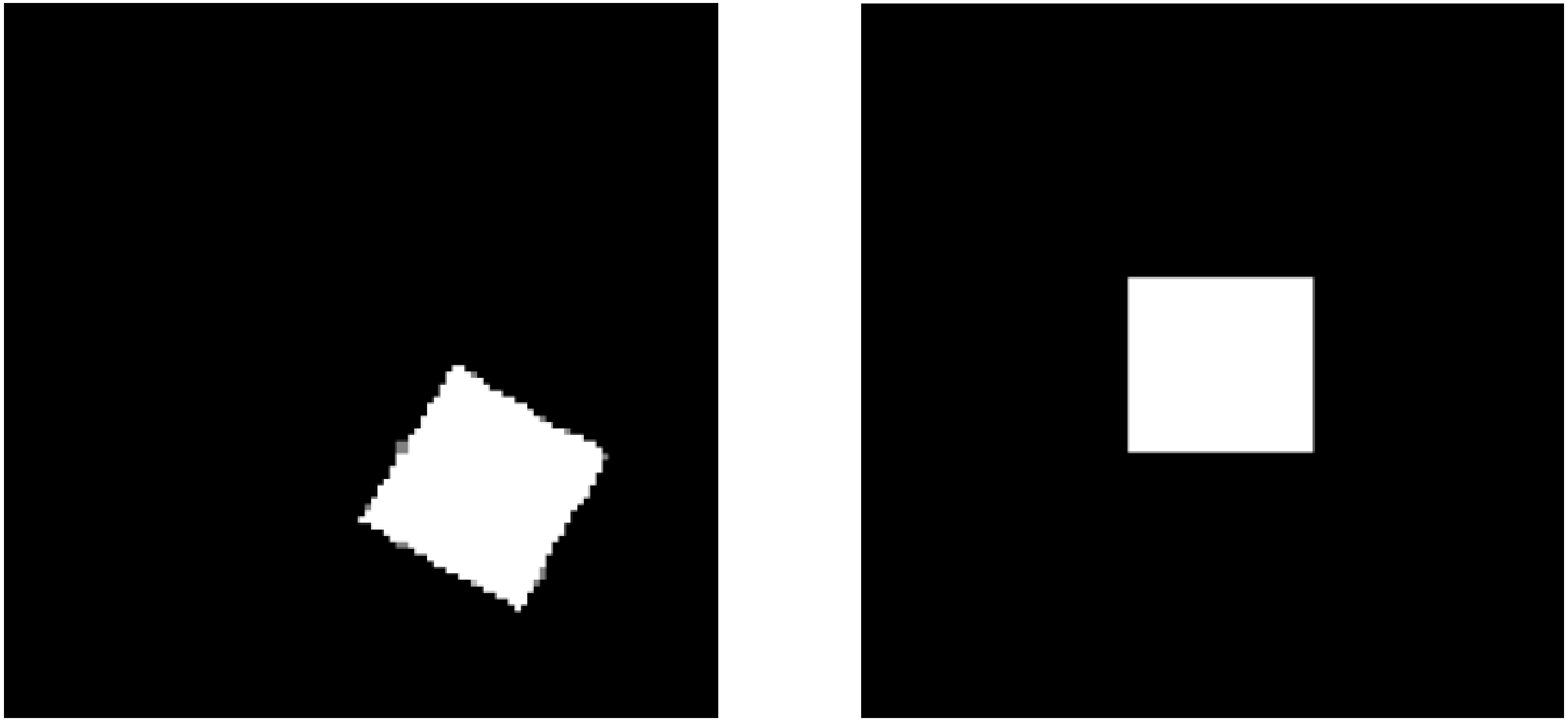
Multi-Resolution



Speeds-up convergence
Helps w/ local minima

Images: ITK Registration Guide

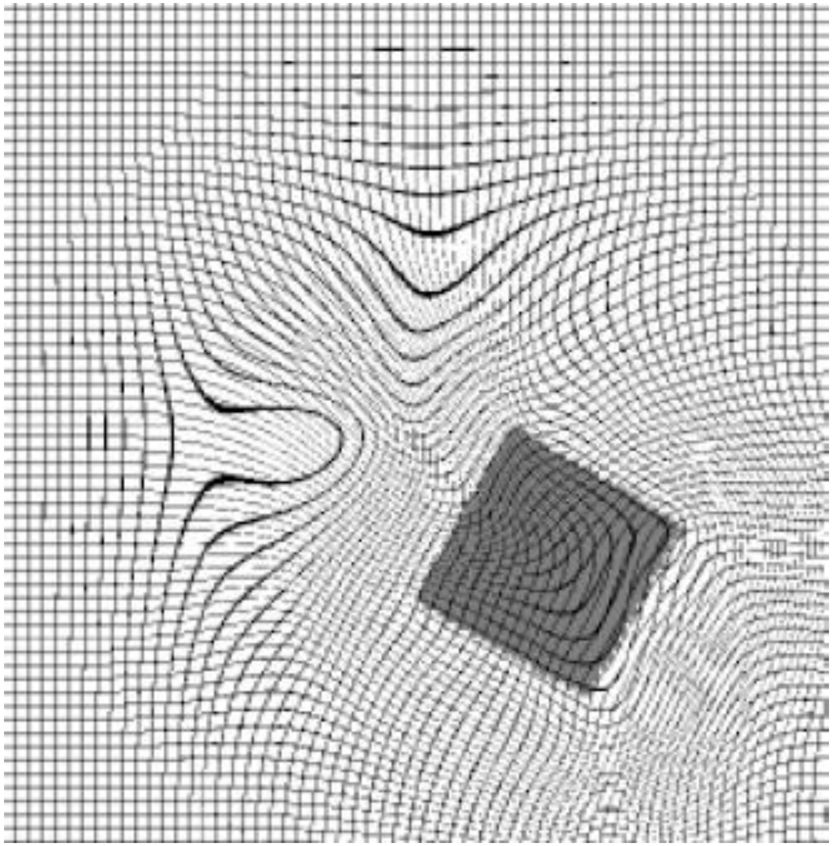
Registration Quality≉Visual Quality

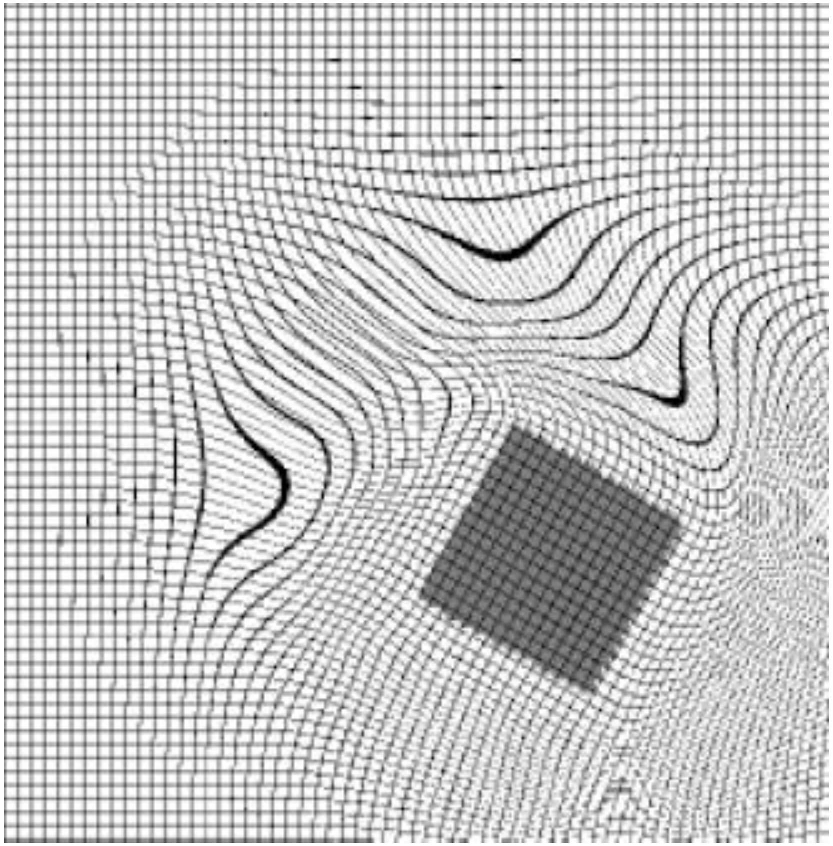


There may be more than one (visually identical) solution, depending on the transformation chosen.

Images: Modersitzki and Haber

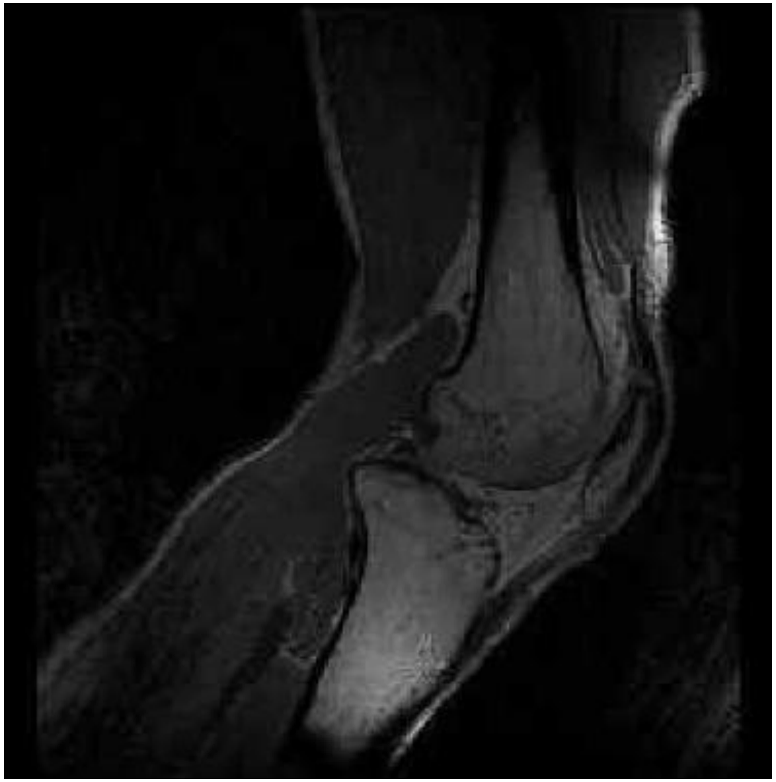
Registration Quality≉Visual Quality





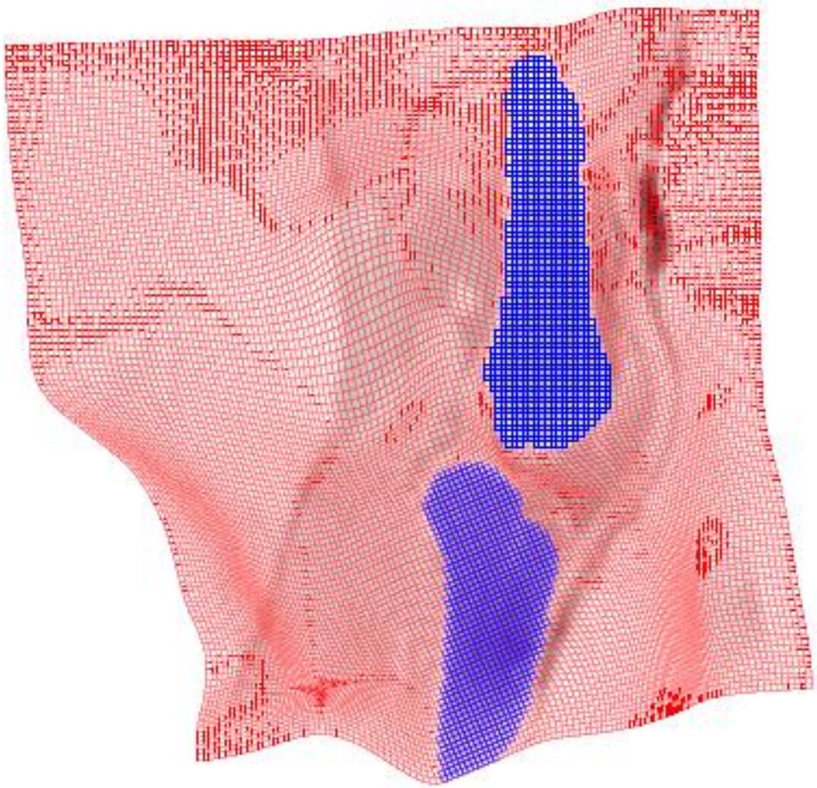
Images: Modersitzki and Haber

Registration Quality$\not\approx$Visual Quality

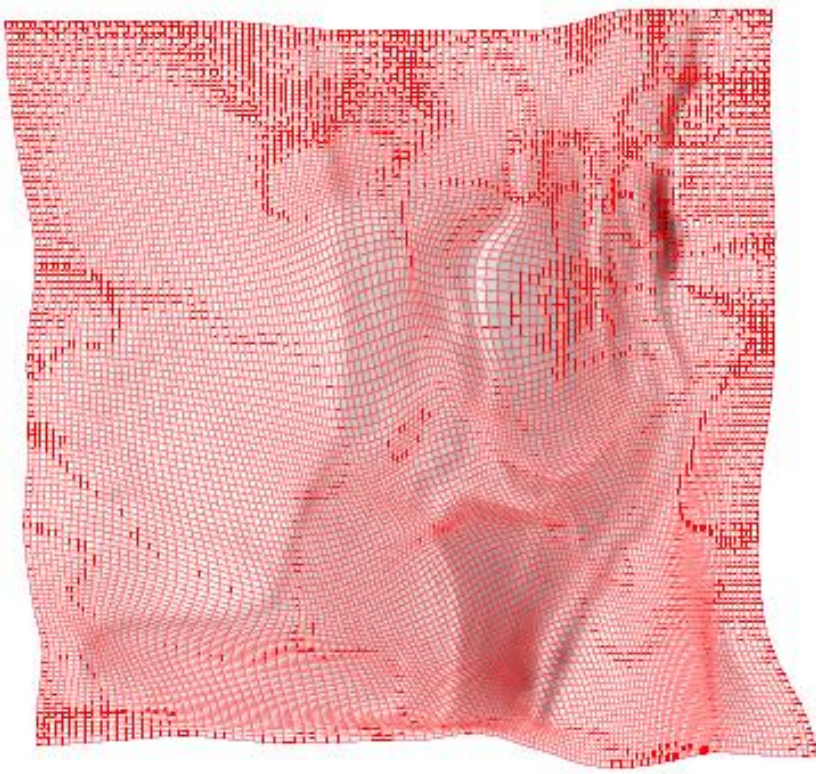


Images: Modersitzki and Haber

Registration Quality≉Visual Quality



With constraint of bone rigidity.



Without constraint.

Images: Modersitzki and Haber

Registration w/ Point Correspondences

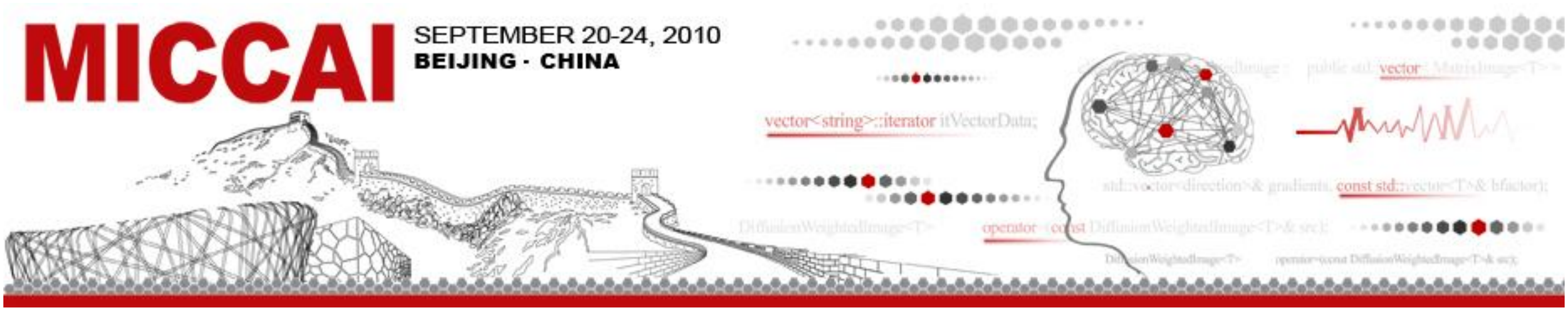
- Landmarks only
  - Sum of
    - Atypicality energy term prevents perfect correspondence
    - Landmark match: typically sum of squared distances
    - No perfect landmark match is achieved
  - Or curved diffeomorphic paths plus piecewise TPS interpolation
- Image registration with point correspondence
  - Image match + landmark match + geom energy
    - No perfect landmark match is achieved
  - Or landmark-constrained registration

Optimization

- By gradient descent on objective function J(T) over space of possible functions T
- So what is the gradient $\nabla_T J$ of a scalar function of a function of typically many variables?
  - Calculus of variations
- With $\nabla_T F$, gradient descent is $\partial I/\partial t = \nabla_T J$
  - Euler-Lagrange optimization

Gradient descent is simplest method. May not have fastest convergence. Other optimization methods exist.

Some Background Material

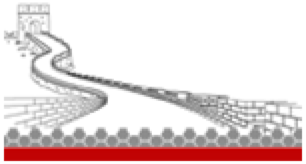


Registration tutorial

- Some of the slides only
- Many details on underlying theory

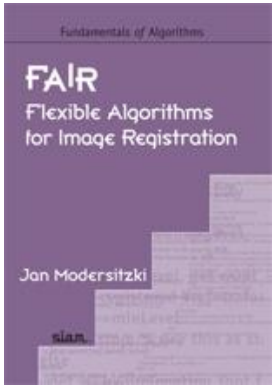
http://campar.in.tum.de/DefRegTutorial/WebHome

INTENSITY-BASED
DEFORMABLE
REGISTRATION
MICCAI**2010**TUTORIAL



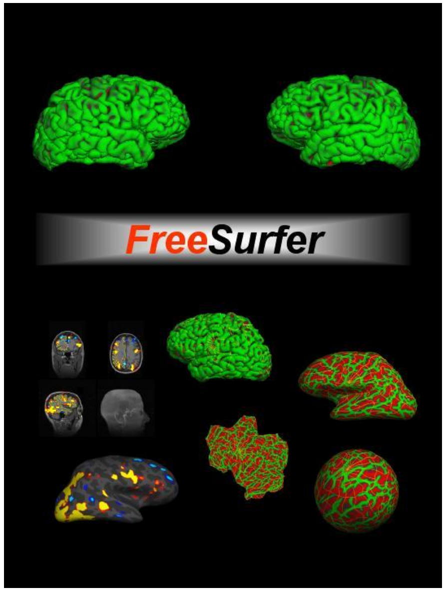
Tools supporting parametric registration

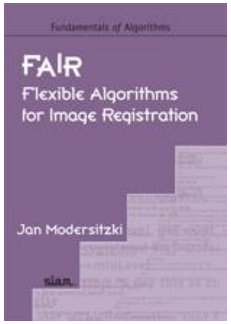
- B-splines
  - Insight Toolkit *itk.org*
  - Elastix *elastix.isi.uu.nl*
  - FAIR *http://www.cas.mcmaster.ca/~modersit/FAIR/index.shtml*
  - Image Registration Toolkit (IRTK) *http://www.doc.ic.ac.uk/~dr/software/index.html*
- Thin-plate splines
  - Elastix



Tools supporting non-parametric registration

- Diffusive Registration
  - FAIR
  - Free-surfer
- Curvature Registration
  - FAIR
- Membrane/Bending Energy
  - FSL *http://www.fmrib.ox.ac.uk/fsl/*
- Demons
  - ITK (Insight Journal)
- Diffeomorphic Demons
  - ITK (Insight Journal)
- Fluid flow
  - FAIR





Tools supporting non-parametric registration

- General diffeomorphic fluid flow
  - ANTs (Advanced Normalization Tools)
    www.picsl.upenn.edu/ANTS/lastix.isi.uu.nl
  - FRAT (Fluid Registration and Atlas Toolkit)
    www.nitrc.org/projects/frat
  - SPM (recommends DARTEL)
    http://www.fil.ion.ucl.ac.uk/spm/
  - AtlasWerks (has GPU implementations)
    http://www.sci.utah.edu/software.html

ANTs

AtlasWerks

3D Slicer

3D Slicer: *slicer.org*

- Integrates a large selection of registration algorithms
- parametric and non-parametric

http://www.slicer.org/slicerWiki/index.php/Slicer3:Registration

Images: slicer.org

3D Slicer registration capabilities

Module	mode		type / DOF of motion								speed			data-type			options			in- & output		
	automated	interactive	rigid (6 DOF)	translation (3 DOF)	similarity (7 or 9 DOF)	affine (12 DOF)	Non-rigid	Non-rigid fiducial	pipelines (affine->nonrigid)	multi-resolution	fast	robust	for Brain ONLY	images	surfaces/models	fiducials	supports ROI mask	supports labelmap mask	choice of cost function	transform input	transform output (affine)	transform output (nonrigid)
Expert Pipeline Registration	•		•		•	•	•		•			•		•				•	•	•	•	
Robust Multiresolution	•					•				•		•		•			•	•			•	
Fast Affine Registration	•					•					•			•						•	•	
Fast Rigid Registration	•		•																	•		
Fast B-Spline Registration	•						•				•			•						•	•	•
Transforms		•	•	•							•	•		•	•					•	•	
Fiducial Registration		•	•	•	•						•	•				•					•	
Surface Registration	•		•		•	•					•				•				•	•	•	
BrainsFit	•		•		•	•	•		•		•	•		•				•		•	•	•
BrainsDemonWarp	•						•							•				•	•			•
BrainsVectorDemonWarp	•						•							•				•	•			•
Plastimatch	•		•	•		•	•	•	•		•			•		•						
Hammer	•						•						•	•								
AC-PC Transform		•	•								•	•	•	•							•	

Large collection of registration algorithms

Images: slicer.org

Questions

Questions?