Compressed Image Quality Evaluation using Power Law Models

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Abstract

This paper presents a compressed image quality measure based on the properties of Zipf law, which is a power-law model adapted from linguistic analysis. It describes the frequency distribution of image patterns, and can be used to put into evidence image details which are affected by compression. Two image quality evaluation measures based on Zipf law have been proposed and tested, one designed to be used on JPEG images and the other on JPEG2000 images. These measures have also been used with the Tabu search algorithm for automatic compression optimization.

1. Introduction

We live in the civilization of image. In our modern civilization, images take an increasingly important place. As images are a type of data which occupies a large storage space, it is necessary to compress the images in order to reduce the data volume both for storage and transmission. However, the only efficient enough compression algorithms are lossy compression algorithms, such as JPEG and JPEG2000, which introduce a loss of information and a distortion on the compressed image with respect to the original image. These algorithms have quantization parameters which can be adjusted to set up the quality and data size of the compressed image. To allow a good adjustment of the compression parameters, distortion measure in the compressed image are needed. In this paper, we will limit us to the particular case of digital photographic images which are intended to be viewed by a human observer. The most obvious method of measuring the perceived distortion is to have the image evaluated by human observers. Different protocols have been proposed for human evaluation, as the MOS scale (Mean Opinion Score) proposed by the CCITT. However, human evaluation is subjective in nature; therefore different persons looking at the same image would have a different perception of its quality.

Moreover, human evaluation is fastidious and time-consuming and cannot be automatic. To solve this problem, many methods for automatic evaluation of compressed image quality have been proposed. Quality evaluation methods can be univariate or bivariate. Univariate, or no-reference methods use only the distorted image, they are useful when the original image is not available. These methods, as those proposed in [1] and [2], are generally usable only to evaluate specific type of distortion on certain image types. Bivariate methods can be full-reference, when the image quality is evaluated using the original image, or reduced-reference when they use only a small subset of parameters extracted from the original image. The most widely used bivariate quality measures are MSE (Mean Square Error) and PSNR (Peak Signal-Noise Ratio). However, they do not always reflect accurately the perceived image distortion. Many other bivariate measures have been proposed, based on human vision models [3,4,5,6]. Delgore et al. [7] have proposed the fusion of different criteria by genetic algorithm. Here, the method described is a reduced-reference bivariate quality measure based on the model deduced from Zipf law, it is a power-law model adapted from linguistic analysis, and it describes the frequency distribution of words in a text. This model provides a global estimation of the distortion of contour zones, which are more pertinent for quality perception than homogenous zones. Since the different compression algorithms do not cause the same type of image distortions, different quality measures can be derived from this model according to the compression type used.

In this paper, we will first present Zipf law and its application to images. Then we will propose image quality evaluation criteria based on Zipf law for both JPEG and JPEG2000 compression. Finally, we will present a compression optimization method using a Tabu search algorithm.

2. Zipf Law
Zipf law is a power law model which has been discovered in 1949 by G.K. Zipf [8] in his works on the distribution of word frequencies in written English texts. According to Zipf law, when in a set of topologically ordered n-tuples of symbols, like words in a text, the distinct n-tuples are ordered in the decreasing order of frequency, the frequency $N_{\sigma(i)}$ of the n-tuple $\sigma(i)$ of rank (i) and the rank itself are related through a power law:

$$N_{\sigma(i)} = k \cdot i^{-\alpha}$$  \hspace{1cm} (1)

In this formula, $k$ and $\alpha$ are constants. The frequency-rank distribution is usually represented in a double-logarithmic scale diagram known as Zipf plot. In this representation of the graph, the plot is linear when the power-law model is verified. This relationship is verified in texts written in any natural language. Several interpretations of this experimental result have been proposed. Zipf’s own interpretation was based on the principle of least effort, while Simon [9] has proposed an explanation based on the properties of stochastic birth and death processes and Mandelbrot [10] another interpretation based on lexicographic trees, which links Zipf law with the notion of fractal dimension. Similar power law distributions have been found in many other domains, for example in economics by Pareto, in urban demography by Gabaix [11], or in Internet traffic by Breslau et al. [12]. In the domain of images, it has been used by Caron et al. [13] for the detection of region of interest in an image.

3. Application to Image

To adapt this model to image study, we need to define an equivalent of the notion of word in a bi-dimensional case. Here we work on image patterns which are defined as 3x3 masks of adjacent image pixels. If the grayscale value of pixels were used directly, the number of different possible patterns would be too high for the statistical distribution to be significant, so we need to cluster the 3x3 pattern elements. Then the patterns are coded in order to reduce the number of distinct patterns. For computing a distortion measure, an image code which preserves the small differences in graylevels between adjacent pixels is needed. The differences are what eyes are sensible to. We have used the general rank method, which consists in replacing the grayscale values of the pixels with their rank in the coding mask. The rank value 0 is affected to the pixel with the lowest gray level value in the mask, and the rank of the other pixels is incremented following the increasing order of grayscale values. The same rank value is affected to every pixel in the mask having the same grayscale value. An example of pattern coded with the general rank method is shown in Figure 1.

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
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<td>210</td>
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<tr>
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<td>34</td>
</tr>
<tr>
<td>40</td>
<td>2</td>
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</tbody>
</table>

**Figure 1.** Original pattern (left) and pattern coded with the general rank method (right)

With the general rank coding, the number of possible different patterns is decreased from $256^9=4.7x10^{21}$ to only about $4.1x10^8$. To apply Zipf law to images, the image is scanned by a 3x3 mask, the patterns are coded using the general rank method and the rank-frequency distribution is represented in double logarithmic scale diagram. We have observed that with digital photographic images, Zipf plot is relatively linear. The most frequent patterns are situated on the main contours of the image, because the homogenous regions of the image are not exactly uniform, small grayscale variations occur randomly. With the general rank coding, they appear to be made of numerous different patterns. On the other hand, contours are made of repetitive patterns which are comparatively much more frequent. When a JPEG compression is applied to the image, the effects of the distortion introduced by the compression are generally more important on contour regions, so this distortion is visible on the Zipf plot associated with a compressed image, as can be seen on Figure 2.

**Figure 2.** Zipf plots associated with an original image (a) and a JPEG compressed image (b)

4. Quality evaluation of JPEG images

A simple way to evaluate the distortion of the Zipf plot associated with a compressed image with respect to the original image would have been to compute the quadratic distance between the two plots. However, it is not sufficient to describe accurately the distortion. Therefore we have designed a quality measure named $ZQ$ (Zipf Quality) which takes into account different parameters that are important for quality evaluation of JPEG images. These parameters are the ratio between the number of patterns that appear more than once on the image and the total patterns number, it expresses the image uniformity, the slope of the plot which characterizes the power law exponent, and the ratio between the frequencies of the most frequent patterns on both images, which represents the amount of repetitive patterns due to compression. Since the
differences between the two plots are more important
in the left part of the plots, the ZQ is computed using
only the M most frequent patterns. The value of ZQ is
given by formula (2):

\[
ZQ' = \frac{-1}{M} \sum_{i=1}^{M} \log_{10}(N_{\alpha_i}) - \log_{10}(N_{\alpha_{i+1}}) \cdot \frac{T/L}{T'/L'} \cdot (P-P') \cdot B
\]  

(2)

In this formula, \(L\) and \(L'\) represent the total numbers
of different patterns, respectively in the original image
\(I\) and the compressed image \(I'\). \(T\) and \(T'\) the numbers of
patterns appearing several times in the image, \(P\) and \(P'\)
are the least-square regression slopes of the two plots
and \(B\) is the ratio between the frequencies of the most
frequent pattern in the two images. This quality
criterion has been tested on 30 grayscale images from
the USC-SIPI database, which are commonly used for
evaluation of image processing algorithms. These
images have been compressed with JPEG using
different values of the JPEG quality factor. Our quality
criterion ZQ has been compared with the PSNR (Peak
Signal/Noise Ratio). To allow a good comparative
criterion ZQ has been compared with the PSNR (Peak
Signal/Noise Ratio). To allow a good comparative
evaluation of results obtained on different images, a
normalized value \(ZQ'\) is computed using formula (3):

\[
ZQ' = \frac{ZQ}{1+ZQ}
\]

(3)

The value of \(ZQ'\) varies in \([0,1]\). This normalized
value allows better comparative evaluation of results
and is more suitable to automatic compression
evaluation. An example of comparative evaluation of
ZQ' and PSNR is shown on Figure 3. Similar results
have been obtained with all the test images. The value
of \(ZQ'\) is close to zero when the compression is low
and the distortion is barely noticeable for the human
viewer, and shows a considerable increase when the
compression deformation becomes visible. Comparatively, the variation of the PSNR is more
linear, and does not show any change of behavior when
the compression starts to be visible. The ZQ' criterion
is therefore more consistent with human perception
than the PSNR.

Figure 3. Evaluation of ZQ' and PSNR (JPEG).

5. JPEG 2000 quality evaluation

The distortion introduced by JPEG2000 is very
different from the one introduced by JPEG
compression: it blurs the image instead of introducing
blocks. This blurring effect causes the homogenous
zones to become completely uniform, which alters
considerably the Zipf plot even when the compression
is not obvious to the human eye. Some parameters of
the ZQ like the regression slope and the distance
between the two plots do not increase with the
compression rate in the case of JPEG2000. In these
conditions, the criterion designed for JPEG images is
not useful for evaluation of JPEG2000 images. We
have proposed another quality criterion which does
not take these parameters into account. It is calculated
according to the formula (4):

\[
ZQ_2 = \frac{B}{L' \log_{10}(N_{\alpha_{i+1}})} \cdot \frac{L}{L'} \cdot \frac{T}{T'} \cdot \frac{P}{P'} \cdot \log_{10}(N_{\alpha_i})
\]

(4)

The results obtained with the ZQ2 are more
consistent with human perception however its variation
is still non-monotonous for small values of ZQ2, due to
the fact the general rank coding is too sensitive to
small gray level variations. Therefore a new coding
method has been used, the thresholded rank coding. It
is computed like the general rank coding, except that
the same rank value is affected to the pixels when the
difference between their grayscale values is inferior to
a threshold which has been set to 10 in our case.

Figure 4. Example of ZQ2 evaluation on a
JPEG2000 image with both coding methods.

An example of result of ZQ2 evaluation is shown on
Figure 4. With the thresholded rank coding, the results
of the evaluation of JPEG2000 image is better in
accordance with the human perception. However, the
criteria based on Zipf law are not suitable for any type
of compression, it is necessary to define specific
criteria for each type of compression.

6. Optimization using Tabu search

The criteria we have defined allow to detect the
value of the compression parameters for which the
compression starts to be visible. We can now use these
criteria to detect automatically the optimal
compression rate for an image which offers the best
compromise between file size and image quality. The
optimization method used is based on the Tabu Search
algorithm. Its principle is to find a configuration of
parameters which minimizes the objective function \(f\)
while memorizing the previously tested configurations.
of parameters in a Tabu list to avoid testing several times the same configuration. From an initial configuration, a list of neighboring configurations, these configurations having parameters values close to the initial values, is randomly generated. The objective function \( f \) is evaluated for these new configurations and the configuration which has the minimal value of \( f \) is retained as the base configuration for the next iteration. The previous configuration changes are stored in the Tabu list and during the generation phase, the configurations in the Tabu list are not retained. The objective function \( f \) is defined according to the formula (6), where \( s \) is the file size in kilobytes and ZQ is the quality criterion determined with Zipf law.

\[
f = \sqrt{\alpha_1 s^2 + \alpha_2 ZQ^2}
\]

Parameters \( \alpha_1 \) and \( \alpha_2 \) take into account the fact that file size and quality criterion do not have comparable numerical scale values. Tabu search optimization has been tested both with JPEG and JPEG2000 compression using ZQ and ZQ\(_\alpha\) respectively. For JPEG images, the only parameter to optimize is the JPEG quality factor. In the case of JPEG2000, we optimize a set of 3 parameters, the bit-rate, the number of resolution layers (C-layers) and the number of wavelet decomposition levels. The values of \( \alpha_1 \) and \( \alpha_2 \) have been determined experimentally in order to get the smaller image file without apparent visual distortion, these values depend on image size. For JPEG images, the values have been set to \( \alpha_3 = 3 \) and \( \alpha_2 = 1.2 \) for 256x256 images and to \( \alpha_3 = 0.3 \) and \( \alpha_2 = 1.1 \) for 512x512 images. For JPEG2000 images they have been set to \( \alpha_3 = 0.9 \) and \( \alpha_2 = 3.5 \) for 256x256 images and \( \alpha_3 = 1.0 \) and \( \alpha_2 = 1.5 \) for 512x512 images. In these conditions, the optimal compression rate found by the Tabu Search method corresponds to an image where the perceived visual distortion is not apparent to the human viewer. The ZQ criteria are well adapted to automatic compression optimization because there is a clear change of behavior between the numerical values of the evaluation criteria whether the compression is apparent or not, which is not the case with other criteria like PSNR.

7. Conclusion

Image patterns, just like words in a text, can have their frequency distribution modeled by a power law. The characteristics of the model describe the complexity of the image texture and, with an appropriate pattern coding method; it is possible to characterize the details of the image which are distorted by image compression. Two distortion measures based on the comparison of Zipf plots associated with original and compressed images have been designed for JPEG and JPEG2000 images. These measures allow to detect the compression rate for which the compression starts to be visible, so they can be used for automatic compression optimization with the Tabu search algorithm. Other multicriteria optimization methods could also have been used, such as genetic algorithms. Complexity measures of image textures based on Zipf law could also be used for other purposes than compression evaluation, for example estimation of the photorealism of synthetic images.

8. References